



MSc in Physics

Top-mass-induced constraints on the Quantum Gravity Scale

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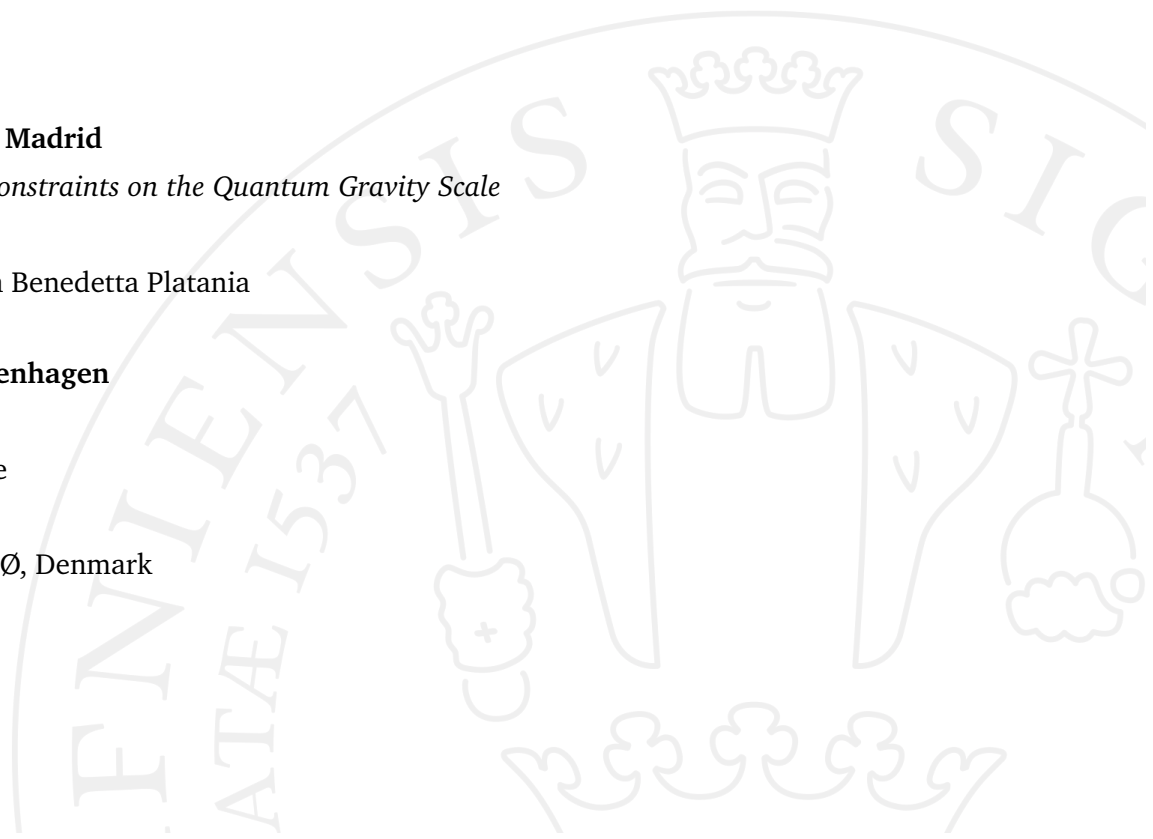
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Abstract

The asymptotic safety paradigm for quantum gravity proposes that gravity can be non-perturbatively renormalizable due to the existence of interacting fixed points that control the flow of the different couplings. It can also be used to provide pre/postdictions for some of the Standard Model parameters, such as the mass of the top quark. This prediction, however, relies on the scale of quantum gravity, which in the context of asymptotic safety is the transition scale to the fixed point regime. This scale is typically set to be the Planck scale. However, in a scenario of "effective asymptotic safety", in which asymptotic safety is not fundamental, but rather a low-energy approximation to some more fundamental theory, the transition scale may not need to be Planckian. The question is then, whether the predictions and postdictions of asymptotic safety remain solid under a change of the fixed point scale. This thesis uses the experimental bounds on the top quark mass to constrain the fixed point scale, showing that this needs to be at least Planckian in order to obtain a top mass compatible with the experiments. We discuss the implications of this finding for a scenario of effective asymptotic safety, setting bounds not only on the fixed point scale, but also on other relevant scales in the context of quantum gravity.

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Introduction

Theoretical physicists aim to describe the fundamental laws of nature in a way as simple and elegant as possible. But the success of a theory is ultimately established by the experimental results, that allow us to test them and assess whether the mathematical and abstract description matches reality or not.

In the last century there were major breakthrough discoveries that changed how we understand physics nowadays. For instance, quantum physics taught us that at microscopic scales some classical notions, such as position, energy, or velocity, are undetermined and it makes no sense to describe them as single-valued functions of time but rather as probability distributions over time and space. On the other hand, Albert Einstein realized that gravity could be seen not as a force, but as a consequence of spacetime curvature. The development of these ideas led to what today are the Standard Model (SM) of particle physics and General Relativity (GR), being both theories capable of making solid predictions that have been widely tested in different scenarios and whose success is proven by an astonishing agreement with experimental and observational data.

So far so good, since we can describe all fundamental interactions that we observe. However, gravity is probably the most peculiar of these. First, we must keep in mind that it is the weakest interaction that we know, at least in the scales that we can test. Indeed, it requires entire astronomical bodies to start noticing its effects, while for the rest of the interactions, we can measure them in laboratories. Second, GR, despite its remarkable predictive power and many tests that confirmed its validity, is not yet embedded in a quantum mechanical framework. We will see in chapter 2 that the standard procedure of quantization, that works perfectly for the rest of interactions, fails when applied to gravity [1]. This is because GR is what is known as a perturbatively non-renormalizable theory, which in short means that predictability is lost for arbitrarily high energies. A way to reconcile this is, for example, by modifying the action in GR to include higher-order terms in curvature, but this in turn would lead to a breakdown of unitarity [2], an important property in any consistent quantum theory. Consequently, a quantum version of GR can only make sense as an effective field theory (EFT), i.e. a theory that is only valid up to a certain cutoff scale, in this

case, the Planck scale. Therefore, it is possible to build a quantum version of GR, the problem is constructing a quantum theory of gravity which is both unitary and ultraviolet (UV) complete.

Knowing this, probably one of the biggest questions in theoretical physics is: Is it possible to formulate a "*Theory of Everything*"? We still do not know how it will look like, provided we ever find it, but for sure it must unify all known interactions, including gravity, in a quantum theory that describes them simultaneously, at all scales. Ideally, it will also solve some issues with existing theories, such as string theory [3], loop quantum gravity [4], and asymptotically safe gravity [5], while potentially revealing new phenomena.

Unfortunately, this work does not pursue the goal of solving the quantum gravity puzzle. Nonetheless, one approach to possibly achieve this, namely, asymptotically safe quantum gravity (ASQG) [6, 7], will be introduced and employed as a framework. This theory seems to provide more predictive power than theories that neglect gravity, as it has been proved in recent works [8–10]. Here, we will use it to find constraints on the quantum gravity scale, i.e. the expected scale at which gravity quantum fluctuations take over and their effects are no longer negligible compared to other interactions.

The idea behind ASQG is that gravity must be well behaved at arbitrarily high energies due to the existence of fixed points that control the renormalization group (RG) flow of the couplings in our theory. Technical details will be explained in the next chapter, i.e. chapter 2, but this highlights the main point of the theory. This approach, unlike others, assumes the usual 4 dimensions of spacetime and the validity of quantum field theory (QFT) as the underlying framework, making it a more conservative proposal. At the conceptual level, ASQG overcomes the problem of the perturbative non-renormalizability of gravity because its UV-completion is an interacting theory, described by a non-perturbative fixed point. The technical novelty is the Functional Renormalization Group (FRG) [11–14], which was introduced in the '90s, and allows us to compute the beta functions that drive the flow of the couplings in an exact non-perturbative way. For that, we need to first choose a truncation scheme, that is, selecting an appropriate approximation for the shape of the action we are considering for our theory. The existence of the asymptotically safe fixed point has been proven within an overwhelming number of approximations [15–23], also in the presence of matter [24–27]. This places the idea of ASQG on solid grounds. Moreover, in recent years more focus has been put on testing unitarity and causality in ASQG, which has so far led to positive results [28–31].

Solving the beta functions that have been derived in simple truncations can give us the running of SM couplings with energy scale [32–34]. The running of couplings depends on the initial conditions of the flow, but the interesting scenario arises when we impose some conditions, namely that the theory is UV-complete, i.e. it has no divergences at high energies. This condition translates directly into constraints on the low-energy interaction couplings—all those that are irrelevant with respect to the UV-completion from which the RG flow departs. In particular, as we shall see in the chapter 3, coupling quantum gravity with matter within the asymptotic safety scenario, enhances the overall predictive power of the theory. This is what has led, e.g., to the prediction of the Higgs mass [8] and the prediction of the top mass [9]. The basis of this work is that the prediction of the top mass depends on which scale one departs from the low-energy EFT, to enter the fixed point regime. This is typically assumed to be the Planck scale, and this is what leads to a top mass that is compatible with experimental data. In this work we challenge this assumption, motivated by possible scenarios of effective asymptotic safety [35, 36], in which ASQG is an intermediate regime between EFT and a more fundamental description, like perturbative string theory. In such a scenario, since the string scale needs to be much lower than the Planck scale, one could only exploit the predictive power of ASQG by lowering the scale at which the theory enters the fixed point regime. In turn, this scale depends on the specific values of the gravitational couplings at the fixed point. Our driving question is *how the experimental bounds on the top mass constrain the quantum gravity scale and hence the possible realization of a scenario of effective ASQG*. Concretely, we will put a lower bound on the scale of the asymptotically safe fixed point, by constraining the fixed point values of the gravitational couplings that are compatible with experimental data. I will also put constraints on the scale of the Landau pole for the subset of RG trajectories that are not UV complete; by adding an arbitrary number of minimally-coupled matter fields, I will show how the Standard Model relates to our predictions, and how to lower the scale of quantum gravity by increasing the number of matter fields. Finally, in chapter 5, I will discuss the implications of this work and the possible scenarios that can also be accommodated while preserving physical sense.

Quantum Gravity and Asymptotic Safety

The starting point should be understanding GR as a classical theory that describes the gravitational interaction. However, unlike other theories, it is not straightforward to construct a quantum version of it just by promoting its fundamental field to an operator and proceed to define observables. This is due to the fact, as we will show, that gravity is a non-renormalizable theory, at least perturbatively. As an approximation, for example when discussing quantum effects on black holes, it is often treated semi-classically, where the background gravitational field is treated classically but the matter fields are quantum. But this of course just simplifies the problem, because there is no reason to believe that gravity should not be treated quantum mechanically as well.

2.1 Gravity as a Quantum Field Theory

Let us first introduce the basic notions of GR and then explain why the quantization procedure fails at the perturbative level. We start from the Einstein-Hilbert action,

$$S_{\text{EH}}[g] = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} (R - 2\Lambda), \quad (2.1)$$

where the couplings present are the Newton's coupling, G_N , defined in terms of Planck's mass as $G_N = M_{\text{Pl}}^{-2} = (1.2209 \cdot 10^{19} \text{GeV})^{-2}$, and the cosmological constant, Λ . The factor $g = \det(g_{\mu\nu})$ carries a negative sign because we are considering Lorentzian signature. If we include an action for the matter fields, S_m , and we perform the variation of the total action with respect to the metric tensor $g^{\mu\nu}$, we obtain the equations of motion as the Einstein's field equations,

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}. \quad (2.2)$$

Here $R_{\mu\nu}$ is the Ricci tensor, that contains second-order derivatives of the metric. It appears also contracted with the metric in the form of the Ricci scalar, $R = g^{\mu\nu} R_{\mu\nu}$. On

the right-hand side, we have the stress-energy tensor, defined in terms of the matter action as

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}}. \quad (2.3)$$

All the information we need to explain the motion of particles or fields in a curved spacetime is encoded in the metric $g_{\mu\nu}$, which is a rank-two tensor. We can count the number of degrees of freedom of this by taking into account certain properties. Symmetry of indices reduces this number from 16 to 10. Then, diffeomorphism invariance eliminates 4 components and Bianchi identities induce constraints on the metric, fixing another four components. Therefore, we are left with only two independent metric components, which correspond to the two physical degrees of freedom, or equivalently, the two possible polarizations of the graviton field.

Now, if we want to construct a QFT out of this action, the standard procedure is to identify the fluctuation field that is going to be quantized. For that, we split the metric into a background metric $\bar{g}_{\mu\nu}(x)$ and a metric fluctuation $h_{\mu\nu}(x)$ in the following way

$$g_{\mu\nu} = \bar{g}_{\mu\nu}(x) + \sqrt{32\pi G_N} h_{\mu\nu}(x), \quad (2.4)$$

such that $\sqrt{32\pi G_N} h_{\mu\nu}(x) \ll 1$, so it is treated as a small perturbation around the background. The numerical factor in front of this field is taken so that $h_{\mu\nu}(x)$ has the usual mass dimension of 1, as corresponds to a boson field. This metric fluctuation, whose excitations will be interpreted as the quantum states populated by *gravitons*, is the field that needs to be quantized. Since the Einstein-Hilbert action is non polynomial, due to the presence of a term like $\sqrt{-g}R$, then we can expand it in powers of h and obtain the interaction terms,

$$S_{\text{EH}}[\bar{g}, h] = S_{\text{EH}}^{(0)}[\bar{g}] + S_{\text{EH}}^{(1)}[\bar{g}, h] + S_{\text{EH}}^{(2)}[\bar{g}, h] + \dots \quad (2.5)$$

The first term is a constant with respect to h and therefore plays no role in the dynamics. $S_{\text{EH}}^{(1)}[\bar{g}, h]$ is proportional to the equations of motion evaluated on the background metric, so assuming it is a solution of these, it has a vanishing contribution. $S_{\text{EH}}^{(2)}[\bar{g}, h]$ is quadratic in h and thus will be the kinetic term from which we can derive the propagator. The higher order terms will be interaction terms, from which we can derive the n -point vertices of gravitons. These terms will have a complicated expression, but schematically will behave as

$$S_{\text{EH}}^{(n)}[\bar{g}, h] \sim \mathcal{O}(M_{\text{Pl}}^{2-n} h^n), \quad n \geq 3. \quad (2.6)$$

This means that there are infinitely many interaction terms, and that the n th term will contain n metric fluctuation fields. Ignoring other possible numerical factors, integrals and tensor structures in these terms, the coupling will correspond to decreasing powers of the Planck mass.

2.1.1 Perturbative non-renormalizability of gravity

The interaction terms given by (2.6) will introduce divergent loop diagrams. In order to determine if a QFT is (perturbatively) renormalizable or not, it suffices to look at the mass dimension of its interaction couplings. Indeed, the superficial degree of divergence, D , of a Feynman diagram G with E external legs and a number of vertices V_n , with $n \geq 3$ legs attached to them is

$$D(G) = d - \frac{d-2}{2}E - \sum_n [g_n]V_n, \quad (2.7)$$

where $[g_n]$ is the mass dimension of the respective coupling and d the number of spacetime dimensions. If at least one of the couplings has $[g_n] < 0$, then $D(G)$ will increase with the number of vertices, leading to an infinite number of diagrams that require renormalization. These theories are (perturbatively) non-renormalizable. Gravity turns out to be such a theory, because if we look at the couplings of higher order terms, $[M_{\text{Pl}}^{2-n}] < 0$ for $n \geq 3$. Therefore, the divergence will become worse as the number of vertices in the diagram increases. This implies that we will need an infinite number of counterterms to renormalize the theory, and thus we lose predictability. Another way to see this is by analyzing the structure of a loop diagram, whose superficial degree of divergence will be [7, 37]

$$D(G) = dL - 2I + 2V = (d-2)L + 2. \quad (2.8)$$

Here, L is the number of loops, and each integral will contribute dL powers of momenta. The number of internal lines is I , and since the graviton propagator behaves like $\sim 1/p^2$, then every line will contribute as $-2I$. Each vertex contains two powers of momentum since the Ricci scalar contains second order derivatives of the metric, therefore contributing as $2V$. In the last equality, we have used the topological identity $L = I - V + 1$. From the expression (2.8), we can see that the superficial degree of divergence increases with the number of loops, proving again that the theory is non-renormalizable.

This is the main problem that GR has when treated as a quantum theory, so while it is a useful and predictive EFT [38–43] for the low-energy regime (i.e. describing the physics that we can measure, such as the motion of planets, deflection of light or even the cosmological evolution), the fact that we inevitably find divergences in the UV is a clear sign that something is missing. There must be a fundamental quantum theory of gravity that is renormalizable, has predictive power and reconciles with the results we have already tested.

Many approaches have been largely studied to tackle this problem [37, 44]. One possibility is to abandon the QFT framework and explore other ideas, such as string theory [45–47] or loop quantum gravity [4, 48]. On the other hand, one can also find ways to fix the problems within a QFT scenario, but that requires sacrificing or reinterpreting some notions, some of them fundamental, for example: in asymptotic safety we simply give up perturbation theory [7], while quadratic gravity [49, 50] needs to redefine unitarity due to the presence of ghost degrees of freedom that propagate on-shell. Hořava–Lifshitz gravity [51] introduces violations of Lorentz invariance, and non-local quantum gravity gives up locality. These are some examples of the many quantum gravity theories that physicists have proposed and studied in recent years.

In the following, I will introduce the basic notions of ASQG and explain the mathematical tools that have been used to develop this work.

2.2 Asymptotically Safe Quantum Gravity

The idea of asymptotic safety was originally proposed by Steven Weinberg in 1979 [6] as a way to explain how gravity can be well-behaved at all energy scales if the RG flow approaches a fixed point in the UV. Weinberg’s proposal, but also in general the most modern understanding of QFT, are deeply rooted in the RG framework developed by Kenneth Wilson and John Kogut in the ’70s [52]. In the Wilsonian perspective, as we shall see in this chapter, a theory is viewed as a point in an infinite-dimensional space of couplings, whose behavior changes with the energy scale under coarse-graining. Fixed points of this flow play a central role: they correspond to scale-invariant theories and define possible continuum limits. In particular, a consistent QFT can be constructed by tuning the microscopic parameters so that the RG trajectory approaches a fixed point as the UV cutoff is removed. When the UV fixed point is at the origin of the theory space, RG trajectories approaching it are asymptotically free, as it is the case of theories like QCD. In essence, asymptotic safety is a generalization of the concept

of asymptotic freedom. The crucial difference is that in asymptotic safety, the value of the couplings at the fixed point does not need to be zero—the UV-completion can be an interacting, and even non-perturbative theory. This leads us to reconsider the notion of renormalization, which is usually closely tied to perturbation theory. By abandoning the possibility of renormalizing the theory only in a perturbative way, a new door opens to find a consistent theory of quantum gravity. The discussion in the following will closely follow the lectures [37].

2.2.1 The concept of Asymptotic Safety

To begin with the discussion in asymptotic safety, let us consider an action in four dimensions, $S = \int d^4x \mathcal{L}[\Phi(x)]$, that depends on different fields that we denote collectively as $\Phi(x)$. Generically, the Lagrangian will contain different interaction terms, all accompanied by their corresponding coupling. Since the mass dimension of the Lagrangian must be equal to the number of spacetime dimensions — in our case four, $[\mathcal{L}] = d = 4$ — depending on the form of the interaction terms and the fields involved in them, the couplings will have different mass dimensions. Consider a coupling $\tilde{\lambda}$ with mass dimension $[\tilde{\lambda}] = d_{\tilde{\lambda}}$. We know a theory will be (perturbatively) non-renormalizable if $d_{\tilde{\lambda}} < 0$, which can be derived from power counting arguments in generic loop diagrams. The divergences are related to the running of couplings, which is often obtained from the RG flow. To do this, we consider the dimensionless version of such a coupling, defined as $\lambda(k) = \tilde{\lambda}(k)k^{-d_{\tilde{\lambda}}}$, such that $[\lambda] = 0$, and we compute its variation with respect to a momentum scale k in the following way

$$\begin{aligned} k\partial_k \lambda(k) &= k\partial_k (\tilde{\lambda}(k)k^{-d_{\tilde{\lambda}}}) = k^{-d_{\tilde{\lambda}}} k\partial_k \tilde{\lambda}(k) - d_{\tilde{\lambda}} k^{-d_{\tilde{\lambda}}} \tilde{\lambda}(k) \\ &= \left(\frac{k\partial_k \tilde{\lambda}(k)}{\tilde{\lambda}(k)} - d_{\tilde{\lambda}} \right) k^{-d_{\tilde{\lambda}}} \tilde{\lambda}(k) = \left(\frac{\partial \log(\tilde{\lambda}(k))}{\partial \log(k)} - d_{\tilde{\lambda}} \right) \lambda(k). \end{aligned} \quad (2.9)$$

We define the *anomalous dimension* of a coupling as

$$\eta[\lambda] = \frac{\partial \log(\tilde{\lambda}(k))}{\partial \log(k)}. \quad (2.10)$$

This has to be at least linear in the coupling, given that in perturbation theory the leading order term in expression (2.9) is given by the term proportional to the “canonical” or classical dimension $d_{\tilde{\lambda}}$. The anomalous dimension encodes all the quantum

corrections that stem from loop diagrams and make the coupling differ from its classical scaling [37]. With this, the general shape of a beta function for a given coupling is

$$\beta_\lambda = k \partial_k \lambda(k) = \left(\eta[\lambda] - d_\lambda \right) \lambda(k). \quad (2.11)$$

If we are close to the free theory, corresponding to $\lambda = 0$, then the anomalous dimension will be negligible, and solving the beta function we can see that the coupling behaves as

$$\lambda(k) \simeq \lambda(k_0) \left(\frac{k}{k_0} \right)^{-d_\lambda}, \quad (2.12)$$

where k_0 is just an arbitrary reference scale. This implies that the mass dimension of a coupling will dictate its behavior close to the free theory. If $d_\lambda > 0$, the negative exponent will drive the coupling towards zero, vanishing asymptotically in the UV. This is the case of an asymptotically free coupling. On the other hand, if $d_\lambda < 0$, then the coupling will increase with the RG scale k , and eventually will diverge. This latter conclusion has a problem nonetheless, since we are assuming the validity of a perturbative expansion in the regime where it clearly does not apply because the couplings becomes too large. Finally, in the case when $d_\lambda = 0$, the coupling is classically marginal, and thus the leading order term will come from the anomalous dimension.

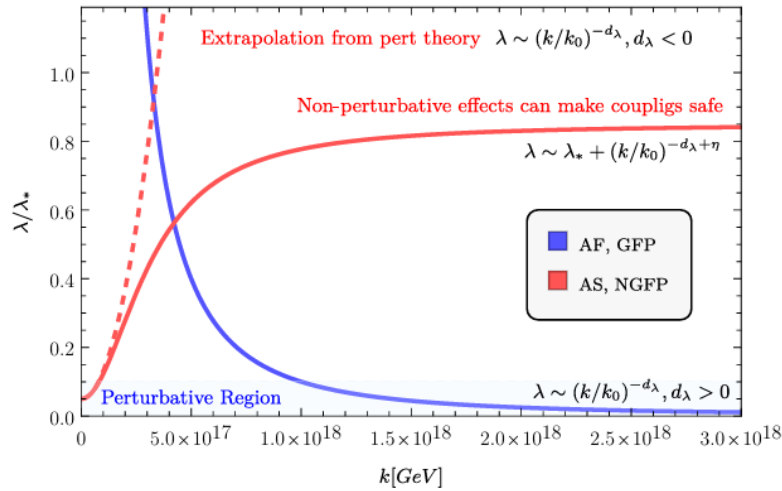


Figure 2.1: Illustration of the ideas of asymptotic safety and asymptotic freedom. Figure taken from [37].

This approximation to the solution is only valid in a perturbative regime, whereas for larger values of the coupling, the anomalous dimension becomes an important contribution and determines the running of the coupling. Effectively, it changes the scaling dimension to $(\eta[\lambda] - d_\lambda)$, and depending on its sign and value, it can bend the

curve that would otherwise grow indefinitely. The anomalous dimension can balance the classical scaling dimension in the non-perturbative regime, potentially making the coupling approach a constant value. In that case, the theory does not become free in the UV, but we prevent the RG flow from divergences. This is precisely what we call an "asymptotic safe" trajectory, and it allows the theory to retain predictability at arbitrarily high scales. This idea is depicted in Fig. 2.1.

2.2.2 RG flow as a dynamical system: fixed points and stability

Having understood the idea of asymptotic safety, let us now elaborate more on the different cases in which this scenario can be realized and the underlying mathematical structure. In a theory space where we have different couplings, labeled with a subscript $n = 1, \dots, N$, the RG flow can be seen as a dynamical system, governed by first order differential equations

$$k \partial_k \lambda_n(k) = \beta_{\lambda_n}, \quad (2.13)$$

in which we can consider the couplings λ_n evolving with the RG time, defined as $t = \log(k/k_0)$. The beta functions are interpreted as vector fields in the theory space driving the flow of the couplings. There are some special points, namely the zeroes of the beta functions, in which this flow stops. These are called *fixed points*, which satisfy

$$k \partial_k \lambda_n^* = \beta_{\lambda_n}(\{\lambda_m^*\}) = 0 \quad (2.14)$$

for all beta functions simultaneously. We denote λ_m^* the fixed point values for each of the couplings. We can classify the fixed points as:

- Gaussian Fixed Point (GFP): $\lambda_n^* = 0, \forall \lambda_n$. All the couplings vanish and the theory becomes free at this point.
- Non-Gaussian Fixed Point (NGFP): There exists at least one λ_n^* for which $\lambda_n^* \neq 0$.

Once we identify the fixed points in our system, it is useful to assess how the flow behaves in a vicinity of those points. For that, we expand the beta functions around a fixed point up to first order

$$\beta_{\lambda_n}(\{\lambda_m\}) \simeq \beta_{\lambda_n}(\{\lambda_m^*\}) + \sum_m \left. \frac{\partial \beta_{\lambda_n}}{\partial \lambda_m} \right|_{\lambda_m = \lambda_m^*} (\lambda_m - \lambda_m^*). \quad (2.15)$$

By definition, the beta functions vanish at any fixed point, so the first term in this expansion must be equal to zero. Rewriting this in a compact way, with $\vec{\beta}$ and $\vec{\lambda}$ being vectors containing the beta functions and the coupling as components, we have that

$$\vec{\beta} \simeq \mathcal{S} (\vec{\lambda} - \vec{\lambda}^*) . \quad (2.16)$$

Here, we defined the *stability matrix* as

$$\mathcal{S}_{mn} = \left. \frac{\partial \beta_{\lambda_m}}{\partial \lambda_n} \right|_{\lambda_i = \lambda_i^*} , \quad (2.17)$$

which can be interpreted as the Jacobian matrix evaluated at the fixed point. Then, the approximate solution valid near a fixed point, obtained by just considering this leading order expansion will be

$$\vec{\lambda}(k) = \vec{\lambda}^* + \sum_i c_i \vec{e}_i \left(\frac{k}{k_0} \right)^{-\theta_i} , \quad (2.18)$$

where $\vec{e}_i$ are the eigenvectors of the stability matrix and θ_i are known as the *critical exponents*, defined as minus the eigenvalues of the stability matrix. The coefficients c_i are just integration constants, and k_0 is an arbitrary reference scale that can actually absorb one of the constants. This solution reminds us in a way of the one given in (2.12), because it is essentially the same idea, just here the mass dimension of the couplings is replaced by the critical exponent, which include the contribution from the anomalous dimension of the couplings and reduce to the canonical mass dimension in the case of GFPs. It is the sign of these critical exponents that dictate the flow of the couplings close to fixed points. If we take the UV limit, $k \rightarrow \infty$, the directions with a positive critical exponent will flow towards the fixed point, while the directions with a negative critical exponent will make the couplings flow away from them. In summary, we will say that:

- $\vec{e}_i$ defines a *relevant direction* if $\theta_i > 0$.
- $\vec{e}_i$ defines an *irrelevant direction* if $\theta_i < 0$.
- $\vec{e}_i$ defines a *marginal direction* if $\theta_i = 0$.

This way, we can classify the fixed points in terms of their critical exponents:

- Fully UV-attractive/IR-repulsive fixed point: $\theta_i > 0 \forall i$, meaning that every eigen-direction is relevant.

- Fully IR-attractive/UV-repulsive fixed point: $\theta_i < 0 \forall i$, meaning that every eigen-direction is irrelevant.
- Saddle point: there are both relevant and irrelevant directions present, consisting of the general case.

Using this analysis, we would be able to sketch how the RG trajectories should look like in a phase diagram. Knowing the fixed points and how the couplings behave when passing close to them, we can build an approximate idea of how the RG flow in theory space will look like. However, the precise solution will clearly require solving the differential equations associated with the beta functions and will depend on the choice of initial conditions.

Another key concept is the *UV critical manifold*, also called basin of attraction, consisting of the set of points in the theory space that flow towards a certain fixed point under the effect of RG transformations in the UV limit. Equivalently, the critical manifold will contain all the asymptotically safe trajectories that approach a certain fixed point in the UV. When we have several fixed points, the different attractive (or repulsive) points will be connected by *separatrices*, which are RG trajectories that connect IR-repulsive points to UV-attractive ones, called sources and sinks, in analogy to a dynamical system.

We will say that a trajectory is *UV-complete* if it lies within the UV critical manifold. Identifying the subset of RG trajectories that are UV-complete amounts to setting to zero all the coefficients c_i corresponding to irrelevant directions, i.e. directions with $\theta_i < 0$. This also means that having irrelevant directions translates into a prediction, since requiring a UV-complete trajectory fixes many of the c_i -coefficients. On the other hand, the coefficients corresponding to relevant directions are free parameters of the theory, and need to be fixed by experiments. Therefore, the number of relevant directions is equal to the dimension of the critical manifold. This is depicted in Fig. 2.2.

Then, a final requirement for a theory to be *non-perturbatively renormalizable* is that the number of free parameters needs to be finite. In this case, we just need a finite number of measurements to uniquely determine the trajectory that is UV-complete. The number of free parameters is equal to the number of relevant directions minus one. This is because one of the coefficients can be reabsorbed in the definition of the reference scale k_0 , and this scale is arbitrary, with it setting the speed of the flow. Another reason of why we can do that, is because we have the freedom to choose a unit

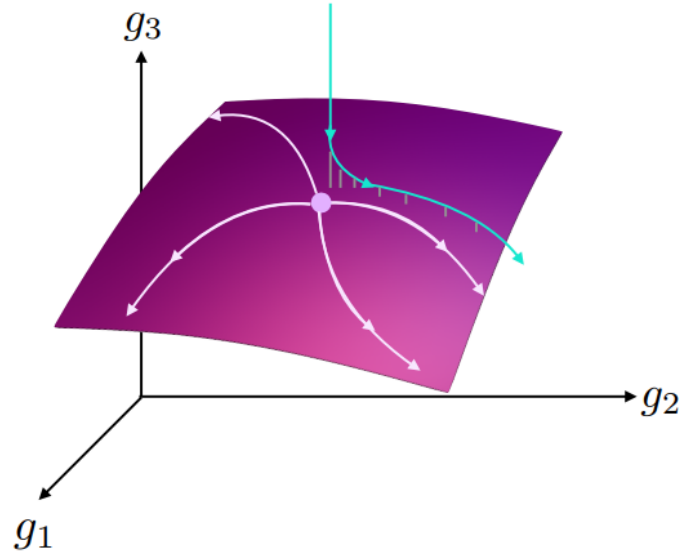


Figure 2.2: Critical manifold spanned by the relevant direction of a certain fixed point. UV-complete trajectories approaching the fixed point must lie within this manifold. Figure taken from [12].

scale of the system, in the same way that the Newton constant is used to define the Planck units. In summary, having a predictive theory, requires the number of relevant directions to be finite.

After having introduced all these ideas, we can now give an answer to how this connects to Weinberg’s proposal of asymptotic safety. We have seen that GR as a perturbative QFT is perturbatively non-renormalizable. However, within the modern, Wilsonian understanding of renormalization, if there exists a non-trivial fixed point with only a finite number of relevant directions, then the theory is non-perturbatively renormalizable. The asymptotic safety paradigm relies on the idea that gravity may make sense as a QFT, if its RG flow is governed by a non-trivial fixed point with a finite number of relevant directions. According to the *asymptotic safety conjecture*, gravity is a non-perturbatively renormalizable theory. Such a conjecture has been corroborated within a large number of different approximations [15–20, 22–24, 53], using both numerical Monte Carlo simulations [54–56], and the FRG, which we explain next.

2.3 Functional Renormalization Group

We now continue the discussion on mathematical tools used in ASQG. For that, we need to introduce the FRG, which is the main technique used to compute the non-perturbative beta functions [11, 12, 57]. We note that this tool is not only used in gravity, rather, it has been corroborated in particle physics and condensed matter, with results close to those of lattice QCD and bootstrap, respectively [12]

In order to capture all the non-perturbative effects, we will employ an *effective action*, denoted as Γ , that includes all quantum corrections. It serves as the generating functional of one-particle irreducible correlation functions, from which we can extract scattering amplitudes that are exact. Let us first comment how to obtain this effective action and what are its main properties. For the derivation, we shall follow the lectures [37] closely.

2.3.1 Effective action

Consider a microscopical or "bare" action $S[\Phi]$ that depends on different types of fields, denoted generically as Φ . These fields can have an arbitrary number of spin, and will therefore have different index structures, but we do not need to write them explicitly for the current purpose. Specifically, we will suppress the corresponding indices in the following notation and assume that they are contracted in an appropriate way. From this action, we can write down the path integral, also called partition function due to its connection with statistical physics, as

$$\mathcal{Z}[J] = \frac{1}{\mathcal{N}} \int \mathcal{D}\Phi \exp \left[-S[\Phi] + \int d^d x J(x) \Phi(x) \right], \quad (2.19)$$

where J is a source term that allows us to obtain the correlation functions by taking functional derivatives with respect to it. Usually, the path integral is formally divergent, but we assume this has been taken care of by the introduction of a cut-off regularization, i.e. a maximum scale Λ_{UV} at which we stop the integration. It is however convenient to use the *Schwinger functional* instead, defined as the logarithm of the partition function

$$\mathcal{W}[J] = \log \mathcal{Z}[J], \quad (2.20)$$

such that the correlation functions can be computed as functional derivatives of it

$$\langle \Phi(x_1) \cdots \Phi(x_n) \rangle = \frac{\delta^n}{\delta J(x_1) \cdots \delta J(x_n)} \mathcal{W}[J] \equiv \mathcal{W}^{(n)}[J]. \quad (2.21)$$

We denote the vacuum expectation value (vev) of a field as:

$$\phi(x) = \langle 0 | \Phi(x) | 0 \rangle = \frac{\delta \mathcal{W}[J]}{\delta J(x)}, \quad (2.22)$$

and we can also write propagators, composed by a connected and a disconnected part, as

$$\mathcal{G}(x_1, x_2) \equiv \mathcal{W}^{(2)}[J] = \langle 0 | \Phi(x_1) \Phi(x_2) | 0 \rangle = \langle \phi(x_1) \phi(x_2) \rangle_J - \phi(x_1) \phi(x_2). \quad (2.23)$$

With this, we can define the effective action as the Legendre transform of the Schwinger functional, where ϕ is the conjugate variable of J

$$\Gamma[\phi] = \sup_J \left\{ \int d^d x J(x) \phi(x) - \mathcal{W}[J] \right\}. \quad (2.24)$$

The quantum equations of motion can be obtained by varying this effective action with respect to the expectation value of the field

$$\Gamma^{(1)}[\phi] = J_{sup} + \left(\phi - \frac{\delta \mathcal{W}[J_{sup}]}{\delta J_{sup}} \right) \frac{\delta J_{sup}}{\delta \phi} = J_{sup}, \quad (2.25)$$

where J_{sup} is the value of J for which the supremum in the Legendre transform in expression (2.24) is reached. If now we compute the second variation, it can be proven that we will obtain the inverse of the propagator

$$\left[\Gamma^{(2)}(x_1, x_2) \right]^{-1} = \mathcal{G}(x_1, x_2). \quad (2.26)$$

This gives the idea of the meaning of an effective action. In order to compute it we will follow the prescription given by the Wilsonian picture of the RG, which consists of integrating out the momentum modes in the path integral "shell-by-shell". We start from higher momenta and perform the integration down to the lower ones, and once the path integral is done, then we obtain the effective action. This is however not easily done, and requires translating the path integral into an integral-functional differential equation, which we introduce next.

2.3.2 The Wetterich equation

To perform the path integral, needed to obtain the effective action, we will introduce the *effective average action* (EAA), Γ_k , for which only modes with physical momentum above a fiducial scale k , i.e. $p > k$, have been integrated out. As a consequence, in the limit $k \rightarrow \infty$ no modes have been integrated out, and therefore all quantum fluctuations are included. This means that we recover the microscopic action, $\Gamma_{k \rightarrow \infty}[\Phi] = S[\Phi]$. In the opposite limit, for $k \rightarrow 0$, all modes regardless their momentum have been integrated out, and thus the full path integral has been performed. In that case, we obtain the effective action as the limit $\Gamma_{k \rightarrow 0}[\Phi] = \Gamma[\phi]$. Then, for any intermediate scale k , we can interpret this average action as an appropriate interpolation between these two limits. This idea is depicted in Fig. 2.3. Our goal in this section is to derive the exact flow equation for Γ_k , known as the *Wetterich equation* [58]. The advantage of this, is that generally it is simpler to solve differential equations rather than performing path integrals.

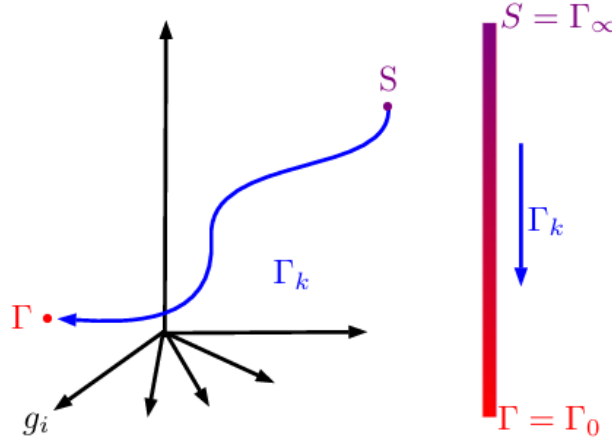


Figure 2.3: Depiction of the FRG flow. The limit $k \rightarrow \infty$ recovers the bare action, while the limit $k \rightarrow 0$ corresponds to the effective action. Figure taken from [37].

To implement the Wilsonian idea of "shell-by-shell" integration, we define a scale-dependent partition function, that includes a regulator term to the microscopic action

$$\mathcal{Z}_k[J] = \frac{1}{\mathcal{N}} \int \mathcal{D}\Phi \exp \left[-\Delta S_k[\Phi] - S[\Phi] + \int d^d x J(x) \Phi(x) \right], \quad (2.27)$$

where the regulator functional takes the following form in momentum space

$$\Delta S_k[\Phi] = \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} \Phi(-p) \mathcal{R}_k(p^2) \Phi(p). \quad (2.28)$$

This regulator acts like a scale-dependent mass term [59], since it is quadratic in the fields. In consequence, it will modify the usual propagator, $\sim 1/p^2$, to one of the form $\sim 1/(p^2 + \mathcal{R}_k(p^2))$. Here, instead of a constant mass, this term will depend on scale, only present for low energy modes.

Given that only momenta with $p > k$ are integrated out in the scale dependent path integral, the regulator should implement this feature. To achieve that, certain conditions must be met, namely:

- Suppression of infrared (IR) modes: $\mathcal{R}_k(p^2) > 0$ for $p^2 < k^2$. This way, the factor $e^{-\Delta S_k}$ suppresses the contribution of low energy modes in the path integral, and thus only modes with $p^2 > k^2$ participate in the integral. A usual choice is $\lim_{p^2 \rightarrow 0} \mathcal{R}_k(p^2) = k^2$.
- In the limit $k \rightarrow \infty$, the suppression is maximal, since we do not want any mode, regardless of their momentum, to be integrated out. Therefore: $\lim_{k \rightarrow \infty} \mathcal{R}_k(p^2) = \infty$. This ensures that we recover the microscopical action in the limit $k \rightarrow \infty$.
- $\lim_{k \rightarrow 0} \mathcal{R}_k(p^2) = 0$ and in general $\mathcal{R}_k(p^2) \rightarrow 0$ for $p^2 > k^2$. This is precisely because we want the high energy modes to be integrated out, so we do not need to introduce any modification in the path integral. This condition ensures that we obtain the effective action in the limit $k \rightarrow 0$.

There is freedom in the shape we choose for the regulator, as long as it satisfies the above conditions. To give some examples, a commonly used one is the Litim regulator

$$\mathcal{R}_k(p^2) = (k^2 - p^2) \theta \left(1 - \frac{p^2}{k^2} \right), \quad (2.29)$$

where θ denotes the Heaviside function. Another possibility is to choose a function that decays in a smoother way, such as an exponential

$$\mathcal{R}_k(p^2) = k^2 e^{-(p^2/k^2)^n}. \quad (2.30)$$

After this clarification on the necessity to introduce a regulator, we come back to the derivation of the Wetterich equation.

In the same way as we did before, we can define the EAA as the Legendre transform of the corresponding scale-dependent Schwinger functional, $\mathcal{W}_k[J] = \log \mathcal{Z}_k[J]$. In this

case, the Legendre transform actually yields $\Gamma_k[\Phi] + \Delta S_k[\Phi]$, so a proper definition for the EAA should be

$$\Gamma_k[\phi] = \sup_J \left\{ \int d^d x J(x) \phi(x) - \mathcal{W}_k[J] - \Delta S_k[\Phi] \right\}. \quad (2.31)$$

From this definition it is clear that we recover the effective action in the limit $k \rightarrow 0$, since $\lim_{k \rightarrow 0} \Delta S_k = 0$. The quantum equations of motion read

$$J_{sup}[\phi] = \frac{\delta(\Gamma_k[\phi] + \Delta S_k[\phi])}{\delta \phi}. \quad (2.32)$$

Finally, the propagator is modified by the introduction of the regulator, as we anticipated

$$\mathcal{G}_k[\phi] = \mathcal{W}_k^{(2)}[J_{sup}] = [\Gamma_k^{(2)}[\phi] + \mathcal{R}_k]^{-1}. \quad (2.33)$$

With this, we can proceed to take the k -derivative of the Γ_k , obtaining

$$k \partial_k \Gamma_k[\phi] = -k \partial_k \mathcal{W}_k[J_{sup}(\phi)] - k \partial_k \Delta S_k[\phi]. \quad (2.34)$$

Let us compute it step by step. First, we will take care of the first term. Knowing that $\mathcal{W}_k[J] = \log \mathcal{Z}_k[J]$, we have

$$k \partial_k \mathcal{W}_k[J_{sup}(\phi)] = \frac{1}{\mathcal{Z}_k[J]} \int \mathcal{D}\Phi (-k \partial_k \Delta S_k[\Phi]) e^{-\Delta S_k[\Phi] - S[\Phi] + \int d^d x J(x) \Phi(x)}. \quad (2.35)$$

Now we make use of the definitions of the regulator in the expression (2.28) and the 2-point correlation function

$$k \partial_k \mathcal{W}_k[J_{sup}(\phi)] = -\frac{1}{2} \int \frac{d^d p}{(2\pi)^d} k \partial_k \mathcal{R}_k(p^2) \langle \phi(p) \phi(-p) \rangle_J. \quad (2.36)$$

For the second term, just using the definition of the regulator we obtain

$$k \partial_k \Delta S_k[\phi] = \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} k \partial_k \mathcal{R}_k(p^2) \phi(p) \phi(-p). \quad (2.37)$$

Now, adding both terms we get

$$k \partial_k \Gamma_k[\phi] = \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} (\langle \phi(p) \phi(-p) \rangle_J - \phi(p) \phi(-p)) k \partial_k \mathcal{R}_k(p^2). \quad (2.38)$$

As a result, if we use the definition of the propagator (2.23), we get

$$\mathcal{G}_k(p, -p) = \frac{\delta^2 \mathcal{W}_k[J]}{\delta J(p) \delta J(-p)} = \langle \phi(p) \phi(-p) \rangle_J - \phi(p) \phi(-p). \quad (2.39)$$

Then the final equation reads

$$k\partial_k\Gamma_k[\phi] = \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} \mathcal{G}_k(p, -p) k\partial_k \mathcal{R}_k(p^2), \quad (2.40)$$

which can be rewritten in a more compact way, taking the usual form of the Wetterich equation [58],

$$k\partial_k\Gamma_k[\phi] = \frac{1}{2} \text{STr} \left[(\Gamma_k^{(2)}[\phi] + \mathcal{R}_k)^{-1} k\partial_k \mathcal{R}_k \right], \quad (2.41)$$

where we used the definition of the propagator in (2.33). Here, STr denotes the "super trace", that includes a sum over discrete indices (gauge, spinorial, spacetime, etc) and an integral over continuous variables (coordinates or momenta).

With this, we have derived an integral-differential equation for the flow of the EAA. Its key features are, that it is exact, since we have not made any approximation, and it is non-perturbative, as we did not perform any perturbative expansion. The choice of regulator does not have any impact on physical observables, unless approximations are made. The advantage that the Wetterich equation provides is that it is a self-consistent equation only involving Γ_k and the regulator, no path integrals need to be performed. Hence, the effective action can be computed directly and the corresponding scattering amplitudes can be computed at tree-level while already including all non-perturbative effects.

2.3.3 Truncation schemes

Now that we derived the Wetterich equation, we have an exact, non-perturbative, integral-differential equation governing the flow of the EAA. This is valid for any theory, as we did not specify the form of the action or the fields involved. The next step would be to find ways to solve this equation. However, in order to be able to solve it, some approximations are necessary. In principle, all terms that are compatible with the symmetries of a system can be generated by quantum fluctuations. Thus, Γ_k will contain an infinite number of terms with their corresponding scale-dependent couplings. In practice, we can only account for a finite number of terms in the computations, and therefore we will need to truncate the action to a finite subset of all possible interactions. This is called a *truncation scheme*, and basically consists of choosing an *ansatz* for the shape of Γ_k .

The running of the couplings that we are considering can be obtained by projecting the left- and right-hand sides of the Wetterich equation onto the corresponding interaction. Then, by comparison of both sides, we can extract the beta functions.

The truncation scheme induces a systematic uncertainty in the results. A way to decrease these uncertainties is by adding more terms, up to a certain *truncation order*, to approximate Γ_k . Prior to choosing a truncation order, we require an *ordering principle* that defines a way to expand the action. The truncation order will then set the accuracy of such approximation. An adequate truncation is usually based on physical intuition about the particular system we are studying. One example of a systematic expansion scheme is the *vertex expansion*, obtained by taking functional derivatives of the action with respect to the fields

$$\Gamma_k[\phi] = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^d x_1 \cdots d^d x_n \Gamma_k^{(n)}(x_1, \dots, x_n) \phi(x_1) \cdots \phi(x_n). \quad (2.42)$$

In this case the truncation order sets the maximum number of fields in the interaction terms present in the action. Yet the action above contains derivatives at all orders. Another common choice is the *derivative expansion*, which includes all orders in the fields, but only a finite number of derivatives,

$$\Gamma_k[\phi] = \int d^d x \left[V_k(\phi) + \frac{1}{2} Z_k(\phi) (\partial\phi)^2 + \mathcal{O}(\partial^4) \right]. \quad (2.43)$$

These expansion schemes are in a way opposite to each other. The vertex expansion accounts for the full momentum dependence and uses the fields as ordering principle. This means that it is non-perturbative in physical momentum p , but the field itself is employed as an expansion parameter. Conversely, the derivative expansion, which is a particular type of operator expansion, orders the term according to the derivative order. This is equivalent in momentum space to an expansion in powers of p . So in this case, the expansion is perturbative in the physical momentum, being only valid for small p , but it is non-perturbative in the fields. In any case, both schemes are non-perturbative in the couplings, meaning that the couplings can take any values.

After choosing an expansion scheme, and setting a truncation order according to the precision required for the problem, one is set to proceed with the calculations. The FRG has been successfully applied to condensed matter physics (with results in line with those from conformal bootstrap), the SM of particle physics, and quantum gravity. See [12] for a comprehensive review of these applications.

2.4 Current state-of-the-art of ASQG

Now that we have reviewed the physics and methodology of ASQG, we close the chapter with a discussion on the main achievements of the field and open questions that are being investigated by the community (see also [5, 13, 59–65] for a comprehensive set of recent reviews on ASQG). Although the techniques presented enable us to obtain solid results, as we will discuss in more detail in chapter 3, the validation of a theory requires passing several consistency tests. These tests consist on checking if fundamental notions, such as UV-completeness (the absence of divergences), unitarity (quantum probabilities being positive and summing up to one) and causality (not having information propagating faster than the speed of light), are satisfied or violated. While the existence of a fixed point, and hence of a UV-completion, stands on solid grounds (see [11, 66] and references therein), there has been criticism on the unitarity of ASQG and on its calculations being based on Euclidean signature [67]. Yet, these questions have been addressed by recent publications [30, 31, 68, 69] and we review this progress below. Besides theoretical studies on consistency conditions, current research in ASQG also focuses on understanding the coupling of quantum gravity with matter [59] (which we will review in detail in the next chapter), scattering amplitudes [70, 71], its implications for cosmology and black-hole physics [64, 72], and its interface with the swampland program [27, 73, 74].

2.4.1 Lorentzian and Euclidean signature

FRG computations are usually performed in Euclidean signature, rather than Lorentzian. The reason for this is because of simplicity in the calculations. However, some quantities, such as scattering amplitudes, need to be computed in Lorentzian signature to represent actual observables. In principle, physical results can be derived by Wick rotating after the computation is done in Euclidean signature [75–79]. This is indeed the standard procedure in QFT. Reconstruction procedures are required in some cases to obtain the Lorentzian results from Euclidean computations. This has been done in particular for scattering amplitudes [29, 70, 71, 80]. Recent works have also made progress in exploring the possibility of computing these quantities directly in Lorentzian signature [29, 81].

2.4.2 Unitarity and causality

Unitarity in ASQG is still an open question. The central question is whether the non-perturbative UV-completion defined by a RG fixed point admits a Hilbert space with positive norm and a unitary time evolution operator, or equivalently, whether scattering amplitudes satisfy the optical theorem. The main reason to ask this question is that ASQG, like many other non-perturbative realizations of quantum gravity, contains infinitely many higher-derivative operators, including curvature-squared terms. In perturbation theory around flat space, truncations of this type lead to propagators with additional poles corresponding to massive spin-2 ghost states, signaling a violation of unitarity. Indeed, by including a finite number of higher-order curvature terms, new poles arise in the propagator when we decompose it in partial fractions. Some of these modes come with negative signs, associated to the negative residues corresponding to ghosts, and violating reflection positivity [82]. According to Ostrogradsky's theorem [83], the violation of unitarity is present, even at a classical level, due to an instability of a theory with a non-degenerate Hamiltonian containing higher-order time derivatives of finite order. However, this type of conclusions only apply when the effective action contains a finite number of derivatives, and this is not the case in ASQG. Finite-order truncations can thus lead to fictitious ghosts which are not in the full theory [28, 31], and hence to misleading conclusions. Testing unitarity in ASQG requires to account for the full momentum dependence of the interaction couplings [65], or, equivalently, form factors [60]. In particular, preliminary studies of the non-perturbative propagator in ASQG show no poles besides the standard graviton (see, e.g., [29, 30]), thus supporting that ASQG is indeed unitary.

One more argument in favor of ASQG being unitary comes from the causal dynamical triangulations (CDT) approach to ASQG, proposed by Renate Loll, Jan Ambjørn and Jerzy Jurkiewicz, see [84]. They found that the transfer matrix connecting spacial slices at different times is self-adjoint, leading to a Hamiltonian in Lorentzian signature that is well-defined and self-adjoint [85, 86]. Assuming that this feature is preserved in the continuum limit, and that a suitable fixed point is found using CDT (which in CDT corresponds to finding a second-order phase transition in the continuum limit), this might signal that ASQG is in fact unitary.

Defining unitarity beyond flat space is also rather tricky: the presence of unphysical degrees of freedom, such as tachyons, ghosts or negative norm states in a given general background does not imply that a theory is inconsistent. It might just be a consequence of that particular background to be unstable [87, 88].

For completeness, let us mention that, while this is not needed in ASQG, there are also other ways to avoid the ghost instability, which are pursued in other approaches to quantum gravity. One of them is by giving up Lorentz invariance, as it is done in Hořava–Lifshitz gravity [51], resulting in a power counting renormalizable and unitary theory of gravity. Another way is by allowing causality to be violated, at least at microscopic scales [89–91], interpreting the negative residues as particles that propagate backwards in time that may not leave any detectable trace.

2.4.3 The entropy puzzle in ASQG

An argument for the impossibility of a QFT of gravity to be UV-complete can be found in [92]. The reasoning is based on the idea that at high energies, the density of states is strongly dominated by black holes. This is called “asymptotic darkness”. We know from black hole thermodynamics that the entropy of a black hole is proportional to the area of the horizon. Then, the asymptotic darkness prediction is $S^{\text{BH}} \propto E^{\frac{d-2}{d-3}}$, while the entropy predicted for a conformal field theory (CFT) scales as $S^{\text{CFT}} \propto E^{\frac{d-1}{d}}$. This mismatch between the density of states indicates that a UV-completion of gravity is not possible within the context of a CFT. In quantum gravity, the area law receives corrections, such as [93, 94]

$$S = \frac{A}{4G} - \frac{3}{2} \log \left(\frac{A}{4G} \right) + \dots, \quad (2.44)$$

which becomes important for smaller black holes. Therefore, it is not clear if the entropy scaling linearly with the area applies in quantum gravity. While the arguments above can be avoided if the fixed point is non-local [74], the question of providing a microscopic explanation for the area scaling in ASQG remains largely open [74].

Some calculations have been made based on model building and RG improvements, see [95–97]. One of the main results is the possibility of the entropy to be compatible with the CFT formula [98] for Planck-sized black holes. Regarding macroscopic black holes, the entropy can certainly be seen as generated by the entanglement of matter fields living on the black hole geometry [99, 100]. All the macroscopic calculations work by constructions, since ASQG reproduced a gravitational EFT at low energies. Yet, a microscopic explanation is structurally not possible, leading to an open puzzle in ASQG [74].

2.4.4 Applications to cosmology and black hole physics

Black hole solutions in ASQG have been mostly studied by incorporating the scale dependence of gravitational couplings into classical solutions, leading to so-called RG-improved geometries (see [64] for a comprehensive review). This procedure captures, in an effective way, the impact of the UV fixed point on spacetime structure, and in particular the fact that in ASQG gravity becomes weaker at very short distances. As a consequence, the classical Schwarzschild singularity is softened and often replaced by a regular core that resembles a de Sitter-like region with finite curvature invariants. The horizon structure is also modified, it being characterized by one inner and one outer horizon. The running of couplings also affects Hawking evaporation, typically slowing it down near the endpoint and supporting scenarios in which evaporation stops at a finite mass. Most results rely on heuristic improvement schemes and are therefore affected by ambiguities in the identification of the RG scale. More recently, a more rigorous approach has been taken, in which black holes in ASQG are directly derived as solutions to the quantum field equations [101–104].

Also in the context of cosmology, implications of ASQG are typically analyzed by incorporating the scale dependence of gravitational couplings into homogeneous and isotropic spacetimes, leading to modified Friedmann dynamics in which Newton's constant and the cosmological constant evolve with the relevant physical scale. This can remove the cosmological singularity [105], in analogy to the case of black holes. Close to the UV fixed point, gravity effectively weakens while the cosmological constant grows, and this behavior can drive an early phase of accelerated expansion without the need for an additional inflaton field, with the expansion arising directly from the fixed-point scaling. For example, in the Starobinsky model, higher-order operators drive inflation instead of an additional scalar field. In the context of ASQG, it has been investigated whether the coupling in the Ricci-squared term, R^2 , can be accommodated accordingly with the Starobinsky inflation to give rise to the observed power spectra [106]. Additionally, there are indications that QG effects drive scalar fields towards flatness [107–109], inducing constraints on the inflationary potential. As the universe evolves toward lower energies, the running of the couplings slows down and classical GR is gradually recovered, providing a continuous connection between a quantum-gravity-dominated early universe and standard cosmology. Within this framework it is possible, under suitable assumptions, to obtain nearly scale-invariant primordial fluctuations, although the precise mechanism and its robustness are still under investigation. General approaches to cosmology of the early Universe explore in what ways ASQG can solve the horizon, flatness and monopole problems, generating at

the same time the expected spectrum of fluctuations without the inclusion of additional degrees of freedom and ad-hoc potentials [110, 111]. In particular, asymptotic safety offers a dynamical perspective on the origin and evolution of the cosmological constant and may lead to modifications of late-time cosmology. Yet, quantitative predictions remain sensitive to the truncation of the effective action and to the identification of the RG scale, and a fully consistent treatment that includes backreaction effects is still an open problem [72].

Gravity coupled to matter

The framework presented in the previous chapter provides the necessary tools to study any non-perturbative QFT. This is particularly useful for studying quantum gravity, given that, as we discussed, perturbation theory fails to fully describe it within a QFT framework. Originally, the proposal of asymptotic safety by Weinberg was conceived as a conjecture. Nonetheless, as we will see, nowadays there is enough compelling evidence to assume that a fixed point indeed exists in different truncations. We now turn our focus to studying not only pure gravity, but also its interplay with matter fields. Although quantum gravity cannot be experimentally tested directly, an indirect way to probe it is by looking at the impact it has on matter, and vice versa, since the presence of matter will also affect the gravitational couplings. We already have a good understanding of matter fields and their fundamental interactions at low energies, since the SM of particle physics has been shown to be an accurate description of them. We now ask ourselves what role gravity can play in the SM, or even in an extension of it, if that is the case in nature. This chapter will closely follow some sections in [59].

3.1 Fixed points in Asymptotically Safe Quantum Gravity

Before discussing the possible ways to couple gravity and matter, we have to say a few words on pure gravity, i.e., on the case in which no matter fields are present. As a starting point, a UV-completion of pure gravity can be found on its own [66], and then one can add matter and gauge fields, to see how the pure-gravity UV-completion is deformed. This strategy can break down if either the gravity-matter system has very different features than the pure gravity case, or if the coupled gravity-matter system is not UV-complete [59]. In general this depends on the matter content, but as we shall see, the matter content of the SM and its small modifications is compatible with an asymptotically safe combined UV-completion [34].

3.1.1 Fixed points in pure gravity

There is strong evidence for the existence of an asymptotically safe fixed point in pure gravity, called the *Reuter fixed point*. This has been originally found in 1996 using FRG techniques for the simplest possible setup: the Einstein-Hilbert truncation [57]. In this case the ansatz for the EAA is chosen to retain the same shape as the action in GR (the action in (2.1)), but with couplings $G_N(k)$ and $\Lambda(k)$ acquiring a dependence on the scale k . For the dimensionless version of these couplings, $g(k) = G_N(k)k^2$ and $\lambda(k) = \Lambda(k)k^{-2}$, the following fixed points have been found, [37]:

- A Gaussian fixed point, $g_* = \lambda_* = 0$ with critical exponents $\theta_{1,2} = \pm 2$. This is a saddle point, and the irrelevant direction is aligned with the direction of the Newton coupling. This implies that, unless we set $G_N = 0$ at all scales, we cannot have a UV-completion that flows towards this point.
- A non-Gaussian fixed point at $g_* \approx 0.541$, $\lambda_* \approx 0.064$, with critical exponents $\theta_{1,2} \approx 2.667 \pm 0.958i$. This point is fully UV-attractive, since the real parts of the critical exponents are both positive, and therefore can prove the existence of a UV-completion in pure gravity. This was found in the first non-perturbative FRG computation for gravity, in [57].
- There is another NGFP in which $g_* < 0$, but we do not consider it simply because that would sit in a regime of repulsive gravity, contrary to every observation. Furthermore, there is a separatrix at $g = 0$ between the regions of positive and negative g , meaning that no trajectory crosses this line. Therefore, fixed points with $g_* < 0$ are disconnected from the physics in our universe.

The robustness of the original computation has been tested using increasingly refined truncations, including higher-curvature terms up to the order 71 in the Ricci scalar, as well as different matter contents, see e.g. [15–27, 53]. In the following, we will see what happens when we include matter fields of different spins, and in particular whether a viable fixed point continues to exist.

3.1.2 Impact of matter fields

In ASQG, we require the existence of a gravitational fixed point that makes the theory renormalizable and UV-complete. The effect of matter fields, including scalars, fermions

and vectors, can potentially alter this fixed point, by either displacing it smoothly or making it disappear completely. In consequence, it is important to analyze the impact of matter fields on this fixed point, and to assess how many degrees of freedom can the theory accommodate without breaking it down. We know that even at a classical level matter has an impact on gravity and vice versa: gravity can be understood as matter "telling" the spacetime how to curve, and spacetime telling the matter how to move. In Quantum Gravity, this picture still holds. In particular, gravity and matter can be:

- Minimally coupled, when the interaction term of matter fields with gravity does not involve any curvature operator. Gravity must be at least minimally coupled, since the factor $\sqrt{-g}$, i.e. the metric determinant, multiplies every term in the general Lagrangian.
- Non-minimally coupled, when interaction terms include the curvature operators, e.g. $R^{\mu\nu}D_\mu\phi D_\nu\phi$, $RD_\mu\phi D^\mu\phi$, $R\phi^2$, $R\bar{\psi}\psi$, $R^{\mu\nu}\bar{\psi}\gamma_\mu D_\nu\psi$.

Therefore, this implies that quantum gravity fluctuations will show up in matter loops, and likewise for matter fluctuations appearing graviton loop corrections. As a result, gravity-matter systems will feature coupled beta functions, affecting each other explicitly. A UV-completion in this case must arise from the interplay of gravity and matter fluctuations, balancing each other to make the full system approach a fixed point.

In a simple setup, setting the cosmological constant to zero and including an arbitrary number of matter fields, the beta function for the Newton coupling takes the following form [59],

$$\beta_g = 2g - g^2 (b_{grav} + a_S N_S + a_F N_F + a_V N_V) + \mathcal{O}(g^3). \quad (3.1)$$

Here, $b_{grav} > 0$ is a term originated from purely gravitational fluctuations, in particular from Fadeev-Popov ghosts contributions [59], and $N_{S,F,V}$ are the number of scalar, fermion and vector fields respectively. We can infer from this that achieving asymptotic safety requires an overall anti-screening contribution from matter fields. By anti-screening we refer to a contribution with a negative sign in the beta function (3.1). This is necessary due to the fact that the canonical scaling of the Newton coupling generates a positive term in the beta function that is independent of the matter content, $2g$, and thus a negative term is required to compensate this. The coefficients $a_{S,F,V}$, which for the case of minimally coupled matter are numerical constants, give information about whether the corresponding fields screen ($a_i < 0$) or anti-screen

($a_i > 0$) the Newton coupling. The gravitational interacting fixed point value that we find is

$$g_* = \frac{2}{b_{grav} + a_S N_S + a_F N_F + a_V N_V} . \quad (3.2)$$

The expression (3.2) shows explicitly the idea we were discussing. We can see that screening contributions ($a_i < 0$) increase the fixed point value, approaching infinity as they approach the value b_{grav} . This destabilize the system and, eventually, if they overcome such value, they can make g_* take negative values. Therefore, screening contributions can remove the gravitational fixed point. On the other hand, anti-screening contributions ($a_i > 0$) drive the fixed point value towards zero asymptotically as we increase the number of fields, favoring the existence of a safe fixed point and making it more perturbative.

Whether matter fields screen or anti-screen the gravitational fixed point is generally determined by its minimal coupling [59]. Recent studies, such as [112], point to the following observations on the different types of fields:

- Minimally coupled scalar fields screen the gravitational coupling. The overall contribution is then negative, $a_S < 0$, increasing the value of g_* .
- Minimally coupled fermions have also a screening effect on the gravitational coupling, $a_F < 0$. The inclusion of a large number of fermions can drastically increase the value of g_* .
- Minimally coupled gauge fields anti-screen the gravitational coupling, $a_V > 0$, stabilizing the fixed point. One thus may expect that the effect of adding more gauge fields makes the gravitational fixed point free. While it is true that $g_* \rightarrow 0$ in the limit $N_V \rightarrow \infty$, the cosmological constant tends to a constant value on the same limit, realizing a gravitational safe fixed point as a consequence.

3.1.3 Standard Model of particle physics

After analyzing the impact of matter fields in a generic way, let us now turn our attention to the matter content of the SM. Because it is a model that we know very well and for which we have performed many tests and measurements, it appears to be a solid candidate to account for the matter fields that exist in nature. We must not forget, however, that it is not yet clear that this is the ultimate model, and indeed possible extensions might be discovered in the future and would need to be considered

if we want to check the consistency of the results. The set of matter fields present in the SM is:

- 4 scalars: the four components of the Higgs doublet.
- 45 Weyl fermions: including leptons, quarks and their color multiplicities.
- 12 gauge bosons: the photon, gluons, $W^\pm$ and Z^0 .

All computations done so far agree on one important result: the asymptotically safe fixed point of pure gravity, namely the Reuter fixed point that we discussed, continues to exist when we account for the matter content of the SM [59]. This is due to the fact that the screening effect of scalars and fermions does not overcome the anti-screening effect of gauge field, making it possible for the Newton coupling to keep a finite positive fixed point value. A further requirement is that the fixed point still exists when we include other gravitational couplings in the gravity-matter system. This first result suggest that ASQG is compatible with the matter content that we observe in our the universe [34]. The fixed point and its critical exponents depend the number of matter fields, and when treated as continuous parameters, the fixed point values in pure gravity ($N_S = N_W = N_V = 0$) can be continuously connected to those of the SM ($N_S = 4, N_V = 12, N_W = 45$), meaning that a combined UV-completion exists [112].

Some extensions of the SM may also admit an asymptotically safe fixed point. Perhaps the most important one, the addition of three right-handed neutrinos, which is necessary to explain neutrino oscillations, does not destroy the fixed point. The same holds for the inclusion of scalar particles, such as axion-like particles, which could provide an explanation for the nature of dark matter. It is then possible to add three additional Weyl fermions and one or two scalars [59]. Models with supersymmetry have also been considered [112, 113].

3.2 Search for a UV-completion of the Standard Model

The SM serves as a good description of the low-energy physics that we can observe. To date, we have only been able to test it up to scales of a few TeV, and although this will probably increase in the following years, reaching the Planck scale seems an

unattainable challenge. Nevertheless, we already know that the SM is just an effective theory, since the model breaks down at highly trans-planckian energies. Therefore, the SM is not UV-complete on its own, but this might change under the impact of ASQG, and it is important to investigate the conditions that could potentially realize this.

Such UV-completion may also provide more predictive power than the SM alone. In an asymptotic safety scenario, if a coupling defines an irrelevant direction for the UV fixed point, its status changes from a free parameter to a calculable quantity. Then, some SM free parameters could be computed from first principles, giving an explanation for open questions in particle physics, such as the origin of mass differences between fermions of the same generation.

3.2.1 Triviality problem

We have seen so far the effect that matter fields have on the gravitational couplings, and how they affect their fixed point. On the other hand, we can discuss how gravitational fluctuations affect the couplings in the SM. For that, let us first introduce the Landau pole problem, also called the *triviality problem*, and how asymptotic safety might serve as a resolution of it.

Consider a generic dimensionless coupling c in the SM. Its RG flow will be governed by its corresponding beta function that will have the following schematic form [59]

$$\beta_c = -f_c c + \beta_{c,1} c^n + \mathcal{O}(c^{n+1}), \quad (3.3)$$

where $n = 2$ for the quartic Higgs self-interaction and $n = 3$ for the gauge and Yukawa couplings. The term $\beta_{c,1}$ is the one-loop matter contribution, present in the SM. The new term we introduce is f_c , which is the gravitational contribution to the beta function of the respective coupling. This structure is general and applies to all sectors of matter couplings; however, the signs of the gravitational and matter contributions may differ in different sectors, leading to different behavior depending on the case.

If we turn off gravity, by setting f_c to zero, we recover the SM beta functions. In that case, couplings with $\beta_{c,1} > 0$ will decrease when the flow moves towards the IR, vanishing in the limit $k \rightarrow 0$, where the theory becomes non-interacting, or trivial. This is what is called the triviality problem. Moreover, if we follow the flow from the IR to the UV, we will find a *Landau pole*, which is a divergence in the coupling at a finite UV

scale. Landau poles in the SM are present for the quartic Higgs coupling λ_H and the Abelian hypercharge coupling g_Y .

The triviality problem is precisely the reason why the SM is not a UV-complete theory but rather an EFT. Its range of validity is limited by the presence of Landau poles, and predictability is lost beyond that scale. The fact that the scale of Landau poles in the SM is highly trans-planckian, $k_{\text{Landau}} \gg M_{Pl}$, leads this problem to be often ignored. A possible solution to this problem may rely on quantum gravity, since this is typically assumed to live at trans-planckian energies. Hence, when coupling quantum gravity with matter, Landau poles can no longer be ignored and instead, one should find a way to UV-complete the theory. Indeed, in the case of ASQG, the interplay of matter with gravity can remove the Landau poles of the SM if the gravitational contribution, i.e. the f_c term, can compensate the growing term $\beta_{c,1}$ [114]. In this way, the unknown physics could be nothing new but gravity, in a strongly coupled quantum version.

3.2.2 Gauge couplings

The gauge interactions in the SM are mediated by vector fields that couple to fermions with different charges. In particular, there are three gauge couplings, associated to three gauge symmetry groups. They are the Abelian hypercharge group $U_Y(1)$, with coupling g_Y ; and the non-Abelian groups $SU(2)$ and $SU(3)$, with couplings g_2 and g_3 respectively. The first two have been unified in a single interaction, in the so-called electroweak unification, under a gauge group $U_Y(1) \times SU(2)$. When this symmetry is spontaneously broken, we recover the electromagnetic $U_{em}(1)$ gauge group, with the usual electromagnetic coupling e , usually in the form of the fine-structure constant, $\alpha = e^2/4\pi$.

We will focus first on the Abelian sector, which presents a problem and a puzzle. The problem consists of the presence of Landau pole in the absence of gravity, i.e. for $f_{g_Y} = 0$. The puzzle is why the coupling takes the specific value in the IR limit $\alpha(k \rightarrow 0) = 1/137$. Asymptotic safety proposes a solution to the problem and the answer to the puzzle, as we will detail in the first part of this section.

Regarding the Landau pole, the charged fermions turn the vacuum into a screening medium, making $\beta_{g_Y,1} = 41/(6 \cdot 16\pi^2) > 0$ [115]. As we discussed before, if the matter

contribution is positive we will encounter a divergence at a finite scale in the UV. At first loop order, the solution to the beta function takes the form

$$g_Y^2(k) = \frac{g_Y^2(k_0)}{1 - 2\beta_{g_Y,1}g_Y^2(k_0) \log\left(\frac{k}{k_0}\right)}, \quad (3.4)$$

k_0 being just a reference scale. This solution is general for a beta function of the type $\beta_g = \beta_{g,1}g^3$, and from it we can see that if the sign of the matter loop contribution is positive, this will lead to a pole at the scale

$$k_{\text{Landau}} = k_0 \cdot \exp\left(\frac{1}{2\beta_{g_Y,1}g_Y^2(k_0)}\right). \quad (3.5)$$

This pole can trivially be avoided by setting $g_Y^2(k_0) = 0$. But this is not a solution, because it would imply that the coupling vanish at all scales, contrary to the observation of a non-zero electric charge at low energy scales. In QED, plugging the corresponding values for the observed fine-structure constant, the loop contribution $\beta_{e,1} = 1/12\pi^2$ and using the rest mass of the electron as a reference scale, we can see that $k_{\text{Landau}} \approx 10^{286}$ GeV. Including two-loop calculations yields as well a trans-planckian energy scale for the pole. Being this a perturbative calculation, the extrapolation to higher values of the coupling should not be trusted. Even non-perturbative methods [116] confirm the existence of such pole in QED. Therefore, we conclude that the physics we know must fail at high energies, above the Planck scale. And since we already know that quantum gravity effects are expected to take over beyond the Planck scale, it is a reasonable candidate to give an explanation of the "new" physics in this regime.

This previous analysis remains true in the absence of gravity, but in order to possibly solve the triviality problem, we must take into account gravitational fluctuations. These will appear in the form of the term f_{g_Y} contributing to the scale dependence of g_Y . Since this term is linear in the coupling, it will dominate at low energies over the pure-matter contribution. This gravitational contribution has been computed using FRG methods [114, 117, 118], and it has been shown that it has an anti-screening effect on the Abelian gauge coupling, i.e. $f_{g_Y} \geq 0$ [119]. This is an expected result, given that in order to achieve a non-trivial fixed point, we need a term with a negative sign that balances the contribution from matter loops. An explicit form for this term, depending on gravitational couplings (in their dimensionless versions) such as the

Newton coupling g , the cosmological constant λ , and higher-order couplings like the coupling b to the term $R_{\mu\nu}R^{\mu\nu}$, has been obtained in [120] and reads

$$f_{g_Y} = \frac{g}{36\pi} \frac{10 + 7b - 40\lambda}{(1 + 2b - 2\lambda)^2}. \quad (3.6)$$

In the UV, at energies above the Planck scale, the gravitational and matter couplings reach their fixed point values. Then, f_{g_Y} should take a constant positive value. At lower energies, the Newton coupling takes its measured value and its dimensionless version then scales as k^2 , decreasing its value towards the IR. Therefore, gravitational fluctuations do not play a significant role below the Planck scale, and will not be noticeable at the usual experimental scales. In consequence, we can neglect this term for $k < M_{\text{Pl}}$, and the scale dependence in this regime is dictated by the SM contribution.

If $f_{g_Y} > 0$, which seems to be the case given the gravitational fixed point values found in the literature [59], the anti-screening effect of the gravitational fluctuation can remove the Landau pole by inducing a safe fixed point at

$$g_Y^* = \sqrt{\frac{f_{g_Y}}{\beta_{g_Y,1}}} \quad (3.7)$$

For small values of the coupling, the Abelian gauge coupling can approach the GFP in the UV, driven by the gravitational contribution. On the other hand, for large values, the SM contribution dominates and the triviality problem persists. However, in an intermediate scenario, when both contributions are balancing each other, the coupling can approach the interacting fixed point (3.7). This is represented in Fig. 3.1.

Furthermore, the fixed point (3.7) has an enhanced predictive power. This can be seen by computing the critical exponent and checking its explicit value, which is indeed negative, $\theta_{g_Y} = -2f_{g_Y}$. Thus, the Abelian gauge coupling defines an irrelevant direction, and a unique value of this coupling in the IR is associated to the unique safe trajectory. This way, we can give an answer to the puzzle of the value of the fine-structure constant by requiring the theory to be UV-completed by asymptotic safety.

The possibility of making a prediction for the Abelian gauge coupling also sets an upper bound for admitted values that avoid the Landau pole. The trajectories satisfying $g_Y(k = M_{\text{Pl}}) < g_Y^*$ will emerge from the asymptotically free fixed point, and are UV-complete. In the opposite case, if $g_Y(k = M_{\text{Pl}}) > g_Y^*$, the triviality problem persists

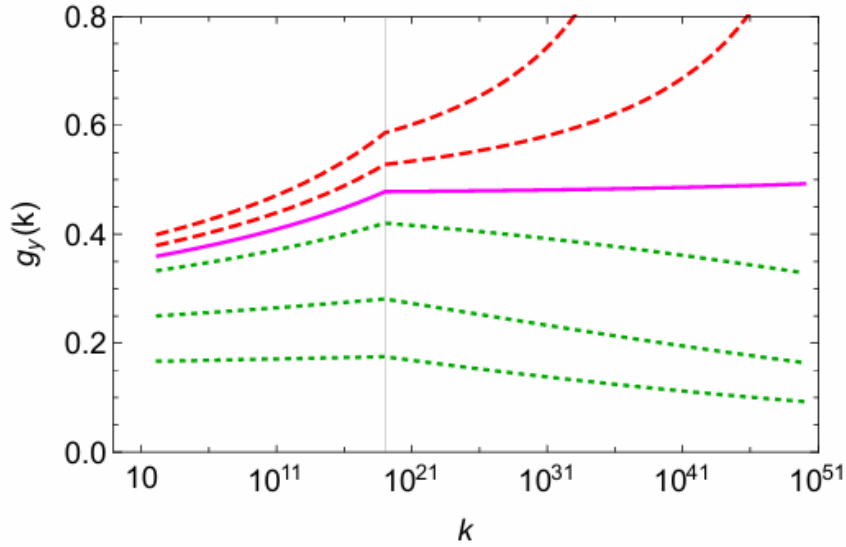


Figure 3.1: Different scenarios for the running of the Abelian gauge coupling g_Y depending on its value at $k = M_{\text{Pl}}$. Figure taken from [59].

as the coupling diverges in the UV. The IR values of the coupling will lie above the fixed point value for the unsafe trajectories, making g_Y^* the upper bound for the prediction from UV-complete trajectories. The case with $g_Y(k = M_{\text{Pl}}) = g_Y^*$ corresponds to selecting the safe trajectory. This bound allows asymptotic safety to become testable, because if it turns out that the predicted value lies below the experimentally measured value for the coupling, then we cannot avoid the triviality problem and a UV-completion is not compatible with observations. Luckily, the measured value of the Abelian hypercharge lies below the prediction [121], and taking into account systematic uncertainties of FRG computations, it may even be in agreement with the prediction. This suggests that asymptotic safety can induce a UV-completion in the Abelian gauge sector, with the predictive power that can answer why the precise value of the coupling in QED is $\alpha = 1/137$ [10, 122].

The non-Abelian gauge sector is simpler due to the fact that the matter contribution in the SM is negative, $\beta_{g_i} < 0$, for the $SU(2)$ and $SU(3)$ couplings. In that case, asymptotic freedom is already present in the theory without gravity, and there is no triviality problem to discuss. Since gravity is insensitive to internal symmetries of the matter sector, at least when coupled minimally, the gravitational contribution is the same for all three gauge couplings. In this way, asymptotic freedom remains a feature of these theories when we include gravity fluctuations [123]. We cannot predict the IR values of the non-Abelian gauge couplings, since they are relevant with respect to the UV fixed point and hence continue to be free parameters.

3.2.3 Yukawa couplings

The Yukawa interactions in the SM give rise to masses of the particles via a symmetry-breaking mechanism. They couple the scalar Higgs field with the different fermions, $y_i \phi \bar{\psi}_i \psi_i$, where y_i is the Yukawa coupling associated to the fermion field ψ_i . Once the Higgs field takes a vacuum expectation value, $\langle \phi \rangle = v$, we can expand it in fluctuations around this value, $\phi = v + \varphi$, φ being the fluctuation fields that we usually interpret as the physical Higgs field. Then, a mass-like term is generated, $m \bar{\psi} \psi$ and we can infer the masses of quarks and leptons from its Yukawa couplings, i.e. $m_i = y_i v / \sqrt{2}$. Given that we have measured the masses of the heaviest quarks [124–126] and heaviest leptons [127] in experiments, we have data on the IR values of the Yukawa couplings.

The beta functions for a simple Yukawa system have the same structure as the gauge sector, i.e. as (3.3) with $n = 3$. The matter contribution is positive, $\beta_{y,1} > 0$, and thus we will encounter a Landau pole in the UV. As we have seen already, this can be removed by asymptotic safety if the gravitational contribution is positive i.e. if $f_y > 0$. The sign of f_y depends on the fixed point value of the cosmological constant, as computed in [128–130]. Then, two scenarios are possible:

- For $\lambda_* < \lambda_{\text{crit}}$, we have that $f_y > 0$. The GFP is UV-attractive. Solutions approaching this point emerge from finite values of the Yukawa coupling in the IR, and those precise values are free parameters. We also find an interacting fixed point, just like the case of the Abelian gauge coupling. This features an irrelevant direction, and an associated safe trajectory, for which we can predict the value of the coupling in the IR. This trajectory enforces an upper bound on the IR value of the coupling, in the same way as we discussed for g_Y .
- For $\lambda_* > \lambda_{\text{crit}}$, we have a screening gravitational contribution, $f_y < 0$ [131]. The Landau problem is still present in this case, and there is no possibility to remove it unless we set $y(k) = 0$ at all scales. This is not compatible with observations in any way, hence we must disregard the possibility that $\lambda_* > \lambda_{\text{crit}}$ (or that a complete theory of gravity with matter can be found solely relying on QFT).

In explicit calculations for a simple Yukawa system, where a single Dirac fermion and real scalar field are taken into account, the gravitational fixed point value lies in the region $\lambda_* > \lambda_{\text{crit}}$ [53, 132], ruling out the possibility of a viable UV-completion in this setup. Including more fields, in particular all fermions present in the three generations of the SM, can shift the gravitational fixed point value to $\lambda_* < \lambda_{\text{crit}}$ [32, 112]. Again,

this favors the idea of a possible UV-completion of the SM plus gravity within the asymptotic safety scenario.

3.2.4 Top-bottom system

We are going to study now the particular behavior of the Yukawa couplings for the heaviest quarks, namely the top and bottom quarks. These, due to their large mass compared to the rest of quarks, will have a higher value for their Yukawa coupling in the IR, and therefore will be the least perturbative couplings. For this reason, it is interesting to study their interplay with gravitational fluctuations.

The Yukawa interaction couple right-handed $SU(2)$ singlets to left-handed $SU(2)$ doublets and the Higgs field. The gravitational interaction, however, is insensitive to any internal symmetry, such as local $SU(2)$ in this case. As a result, the gravitational contribution to the beta function of Yukawa couplings is the same within a $SU(2)$ doublet. This means that f_y for the top and bottom Yukawa couplings is the same in both cases.

If the scenario where $\lambda_* < \lambda_{\text{crit}}$ is realized, then both couplings can be asymptotically free. They cannot be both asymptotically safe at the same time, because then they would share a fixed point value, leading to a similar mass if we follow the trajectories into the IR. This does not correspond to the experimental fact that the top mass is approximately forty times larger than the bottom mass. This is only true neglecting the contribution from the gauge couplings. The top and bottom quarks have different electric charges, hence they have different $U_Y(1)$ hypercharges. Therefore, the contribution from the Abelian gauge coupling g_Y will be different for each quark, making $y_t^* \neq y_b^*$ as long as $g_Y^* \neq 0$. This is reflected in their beta functions, which have the following shape [122]

$$\beta_{y_t} = \frac{y_t}{16\pi^2} \left(\frac{9}{2}y_t^2 + \frac{3}{2}y_b^2 - \frac{17}{12}g_Y^2 \right) - f_y y_t, \quad (3.8)$$

$$\beta_{y_b} = \frac{y_b}{16\pi^2} \left(\frac{9}{2}y_b^2 + \frac{3}{2}y_t^2 - \frac{5}{12}g_Y^2 \right) - f_y y_b. \quad (3.9)$$

The interacting fixed point that arises from these beta functions satisfies the relation

$$y_t^{*2} - y_b^{*2} = \frac{1}{3}g_Y^{*2}, \quad (3.10)$$

proving the previous statement. As a consequence, the fixed point value for the top Yukawa coupling is always larger than the one for the bottom quark. Then, choosing appropriately values for f_g and f_y and solving simultaneously the beta functions for the gauge and Yukawa couplings, we can reproduce the IR values of all couplings that closely correspond to the ones in the SM [9, 34].

The detailed mechanism establishes a connection between the charge ratio of the top and bottom quarks to their mass ratio. Changing the charge ratio would lead to different masses for the top and bottom quarks. In calculations that employ the previous beta functions [33, 122], the values are not not entirely compatible with measurements, and the top quark mass predicted as a result is around 10 GeV heavier. This, however, must be understood to contain uncertainties due to neglected higher-order corrections and systematic errors induced by FRG computations.

3.2.5 Higgs quartic coupling

It was shown in [8] that the ratio of the Higgs boson mass to its vacuum expectation value is a prediction in ASQG. This result is even more remarkable if we take into account that it was predicted before the first detection of the Higgs boson at the LHC [133]. This predicted value directly depends on the Abelian gauge coupling and the top Yukawa coupling. In the case where both are asymptotically free, the resulting Higgs mass is just a few GeV above its experimental value.

Here, we present the beta function associated to the Higgs quartic coupling [59]

$$\begin{aligned} \beta_{\lambda_H} = & -f_s \lambda_H + \frac{1}{16\pi^2} \left(-6y_t^4 + \frac{3}{8} \left(2g_2^4 + (g_2^2 + \frac{5}{3}g_Y^2)^2 \right) \right) \\ & + \frac{1}{16\pi^2} \lambda_H (12y_t^2 - 9g_2^2 - 5g_Y^2) + \frac{3}{2\pi^2} \lambda_H^2. \end{aligned} \quad (3.11)$$

In principle, this should also include the contribution from all other Yukawa couplings. In practice, however, these are negligible compared to the top quark contribution, because it is the heaviest of all fermions.

As argued in [8], if the gauge couplings and the top Yukawa coupling are asymptotically free (under the impact of quantum gravity), and the gravitational contribution $f_s < 0$, then the GFP, $\lambda_H^* = 0$, is IR-attractive, enforcing $\lambda_H(k = M_{Pl}) = 0$. In fact, the results of LHC show that $\lambda_H(k = M_{Pl}) \approx 0$ is needed to obtain the measured Higgs boson mass. Giving an explanation for this is still an open question for particle physicists. Below the

Planck scale, we have a non-vanishing coupling generated by the matter fluctuations coming from the terms y_t^4 and g_Y^4 in (3.11). The corresponding trajectory links the free fixed point with a uniquely determined value of the coupling at the electroweak scale. Thus value determines the ratio of the Higgs mass m_H to the Higgs vacuum expectation value,

$$\lambda_H(k_{\text{IR}}) = \frac{1}{2} \left(\frac{m_H}{v} \right)^2. \quad (3.12)$$

This prediction is sensitively dependent on the top mass [134], which comes with systematic uncertainties. For example, a top mass of $m_t = 172.9$ GeV yields a Higgs mass of $m_H = 129$ GeV. But also, a top mass of $m_t = 170.9$ GeV, still within experimental bounds, yields a Higgs mass of $m_H = 126$ GeV, compatible with $\lambda_H(k = M_{\text{Pl}}) = 0$.

These arguments led to a successful prediction of the Higgs mass near the measured value [133]. The Higgs sector has a special position among the rest of sectors of the SM. This is because, although we can give predictions of parameters from first principles, they were only given after they had been measured. These are technically *post-dictions*, not really predictions.

The prediction on the Higgs mass based on the fixed point $\lambda_H^* = 0$ only depends on the sign of f_s , since it does not appear on the fixed point value. Nonetheless, it does appear on the expression for its critical exponent, and only if $f_s < 0$ the fixed point is IR-attractive. There is, indeed, evidence for f_s being negative [109, 135].

There is another possibility in which an interacting fixed point for λ_H serves as a UV-completion for the Higgs sector. In this case, the Abelian gauge coupling and the top Yukawa coupling need to attain interacting fixed-point values, and the fixed-point value λ_H^* depends explicitly on f_s , not just on its sign. However, if we make the gauge and Yukawa couplings take fixed-point values compatible with their IR values, then the resulting Higgs mass comes out above 129 GeV, ruling out the possibility of using this fixed point to give a solid prediction for the Higgs mass. The only option to reconcile the prediction with experimental data in this case is to include scalar particles [136] that could also potentially explain the nature of dark matter.

Top mass from Asymptotic Safety

In this chapter, I will show in detail how a post-diction for the top quark's mass can be drawn from an asymptotic safety scenario. This calculation is not new, rather, this chapter reproduces the results in [9], but sets the stage for its generalization. Before proceeding, we first need to set up the problem we are going to discuss. In the process, some questions may naturally arise, such as: Why the top quark? Why not the mass of any other particle or any other parameter? Is it possible to always predict something? And if that would be the case, how does this reconcile with the SM of particles that we know? This, and other issues, will be discussed along the chapter and some insights will be provided as well as detailed computations.

While this chapter aims at reproducing the results of [9], we shall set up the calculations in a more general way. In this way, we pave the way to answering the physical questions on the scales of quantum gravity, which will be the subject of the next chapter. We will start this chapter with a novel, general analysis of gravity-matter beta functions, and we will only specialize to the case of [9] in the very last sections 4.5 and 4.6, where, assuming the quantum gravity scale is of the order of the Planck scale, we will reobtain the value of the top mass post-dicted in [9].

4.1 Motivation: the different scales in quantum gravity

In the context of ASQG, a central question is whether the Reuter fixed point is the ultimate UV-completion, or whether it only governs an intermediate effective regime of gravity and matter, connecting the low-energy EFT with a more fundamental description, such as string theory. Such a scenario goes under the name of *effective ASQG*. If the fixed point is fundamental, the RG flow of the dimensionless couplings $\alpha_i(k)$ approaches a UV fixed point α_i^* as $k \rightarrow \infty$, with $\beta_i(\{\alpha_j^*\}) = 0$, so that the theory is UV-complete and predictive in the Wilsonian sense, with a finite-dimensional critical

surface controlling all high-energy physics. In the effective interpretation of ASQG, the RG flow is attracted towards the vicinity of the Reuter fixed point over a large range of scales, leading to approximate scale invariance and enhanced predictivity, but eventually the flow departs from it when new degrees of freedom associated with the fundamental theory become relevant. In this sense, effective asymptotic safety acts as a bridge: it governs the gravitational dynamics in a universal EFT-like regime while encoding partial information about the underlying UV-completion without requiring that the fixed point itself is fundamental.

In this scenario, there would be a non-trivial interplay of scales, whose precise ordering and quantitative values may affect some of the main predictions of ASQG:

- The fixed point scale, k_{FP} , which is the transition scale to the fixed point regime and what we will call the scale of quantum gravity. For $k > k_{\text{FP}}$, gravitational fluctuations become relevant and can potentially drive couplings to a fixed point. This scale will be dictated by the behavior of the gravitational couplings.
- The scale k_{QFT} at which a potential non-QFT theory, such as string theory, kicks in. We cannot extrapolate any result beyond this scale using QFT techniques. A possible candidate for this is the string scale, M_S , which is related to the extended nature of strings relative to particles [37]. When strings are weakly coupled, $M_S \ll M_{\text{Pl}}$, and new physics emerges much earlier than expected.
- The scale of the Landau pole, k_{Landau} , namely the scale at which QFT breaks down. From the viewpoint of the non-perturbative RG, this is the scale at which unsafe RG trajectories diverge. The solution cannot be computed for $k > k_{\text{Landau}}$, as the trajectories are not defined for energies higher than Landau poles, signaling a break down of the theory. Therefore, this will impose an upper bound on the QFT scale, $k_{\text{QFT}} \leq k_{\text{Landau}}$.

Based on the observation that most of the predictions in ASQG rely on a Planckian fixed point scale, *the scope of this thesis is to bound and quantify the relative magnitude of the scales above, using theoretical consistency and compatibility with experimental data.*

4.2 Running gravitational couplings and the fixed point scale

In this section, we introduce two approximations for the running of the gravitational couplings in the Einstein-Hilbert truncation and explain how they are crucially related to the fixed point scale k_{FP} .

In a pure gravity system, the beta functions for the dimensionless Newton's and cosmological constant take a very simple form [66]. Although the fixed point values coming out of calculations are specific numbers, we can recast the beta functions of [66] in a more general form that allows for generic fixed point values,

$$\beta_g = 2g \left(1 - \frac{g}{g_*} \right), \quad (4.1)$$

$$\beta_\lambda = -2\lambda + 4\frac{g\lambda_*}{g_*} - 2\frac{g\lambda}{g_*}, \quad (4.2)$$

where g_* and λ_* are generic fixed point values for the gravitational couplings, which we can later treat as external parameters. Based on this, we can extract two approximations for the running of the dimensionless Newton's and cosmological constants as exact solutions of the beta functions presented above, which are also in very good agreement with full numerical results [81]. The approximation for the Newton coupling reads

$$g(k) = G_N(k)k^2 = \frac{G_{N,0}k^2}{1 + g_*^{-1}G_{N,0}k^2}, \quad (4.3)$$

where $G_{N,0} = G_N(k=0) = M_{\text{Pl}}^{-2}$ is the IR value of the Newton coupling and corresponds to its experimentally determined value. This approximation is constructed to satisfy some of the expected limits in the IR and in the UV, namely

$$\lim_{k \rightarrow 0} G_N(k) = \lim_{k \rightarrow 0} g(k)k^{-2} = G_{N,0}, \quad (4.4)$$

$$\lim_{k \rightarrow \infty} g(k) = g_*. \quad (4.5)$$

The running cosmological constant instead reads

$$\lambda(k) = \Lambda(k)k^{-2} = \frac{\Lambda_0 g_* + \lambda_* G_{N,0} k^4}{G_{N,0} k^4 + g_* k^2}. \quad (4.6)$$

This relation also satisfies some expected limits, such as $\lim_{k \rightarrow 0} \Lambda(k) = \Lambda_0$ and $\lim_{k \rightarrow \infty} \lambda(k) = \lambda_*$. In the case when we consider a vanishing value for the dimensionful version of the cosmological constant in the IR, i.e. $\Lambda_0 = 10^{-120} M_{Pl}^2 \approx 0$, this expression simplifies to

$$\lambda(k) = \frac{\lambda_* g_*^{-1} G_{N,0} k^2}{1 + g_*^{-1} G_{N,0} k^2}, \quad (4.7)$$

resembling the form of (4.3). These are the approximations we are going to use for the following calculations. The running of the gravitational couplings is displayed in Fig. 4.1.

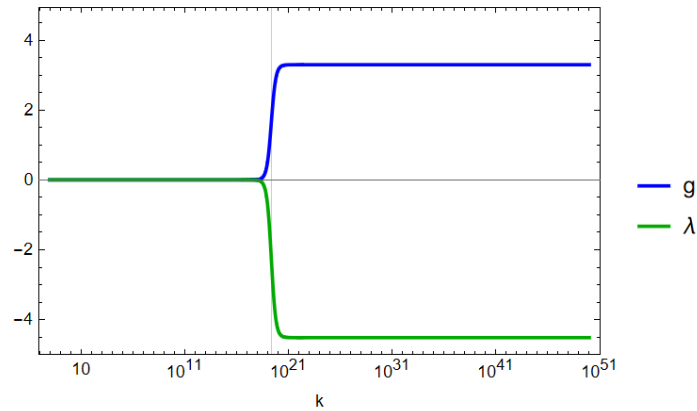


Figure 4.1: Approximations for the running of the gravitational couplings in the case when we account for the matter content of the SM. The RG scale k is measured in GeV.

4.2.1 Quantum gravity scale as fixed point scale

Based on the approximations above, we are now ready to extract the relation between the fixed point values of the gravitational couplings and the fixed point scale, defined as the transition scale to the fixed point regime. This is easily seen by expanding the expression (4.3) around $k \rightarrow \infty$,

$$g(k) = g_* \left(1 - \frac{g_* M_{Pl}^2}{k^2} \right) + \mathcal{O} \left(\frac{1}{k^3} \right). \quad (4.8)$$

This way, we can identify a transition scale at which we enter into the fixed point regime,

$$k_{FP} = \sqrt{g_*} M_{Pl}. \quad (4.9)$$

Then we can rewrite the expression (4.3) in terms of k_{FP} as

$$g(k) = \frac{g_* \left(\frac{k}{k_{\text{FP}}}\right)^2}{1 + \left(\frac{k}{k_{\text{FP}}}\right)^2}. \quad (4.10)$$

Similarly, for the cosmological constant, we can employ a similar approximation starting from the relation (4.7), obtaining

$$\lambda(k) = \Lambda(k)k^{-2} = \frac{\lambda_* \left(\frac{k}{k_{\text{FP}}}\right)^2}{1 + \left(\frac{k}{k_{\text{FP}}}\right)^2}. \quad (4.11)$$

This expression shows explicitly that our first guess for the quantum gravity scale was accurate: the quantum gravity scale of ASQG, intended as the transition scale to the fixed point is universal, and only depends on the fixed point value of the Newton coupling. This is precisely because in the approximations we used which, we remark, are highly accurate, the beta function for g does not depend on λ , even though the converse is not true. We can also notice that k_{FP} corresponds to the scale at which these couplings reach half of their fixed point value, i.e. $g(k_{\text{FP}}) = \frac{1}{2}g_*$ and $\lambda(k_{\text{FP}}) = \frac{1}{2}\lambda_*$.

4.2.2 Gravitational beta functions from gravity-matter systems

Although for most of our calculations and for the scope of this thesis we shall use the parameterizations above, the original calculation of the top mass [9] used specific beta functions for the gravitational couplings, which have been computed in [112] by starting from an effective action of gravity in the Einstein-Hilbert truncation minimally coupled with an arbitrary number of free scalar, Weyl fermion, and vector fields, in the Landau gauge. They read

$$\beta_g = 2g - g^2 f_G(\lambda), \quad (4.12)$$

$$\beta_\lambda = -2\lambda - \frac{g}{2\pi} \left(7 - \frac{3}{2(3-4\lambda)} + N_W - \frac{N_S}{2} - N_V - \frac{5}{2(1-2\lambda)} - 8 \cdot \ln(3/2) \right) - g\lambda f_G(\lambda), \quad (4.13)$$

$$f_G(\lambda) = \frac{5}{6\pi(1-2\lambda)} + \frac{5}{3\pi(1-2\lambda)^2} - \frac{1}{2\pi(3-4\lambda)} + \frac{11 + 32 \log(3/2)}{12\pi} - \frac{N_W + N_S - 4N_V}{6\pi}. \quad (4.14)$$

If we take as input the number of fields in the SM , i.e. $N_S=4$, $N_V=12$, $N_W=45$ as number of scalars, vectors and Weyl fermions respectively, we can find the following interacting fixed point

$$g_* = 3.29, \quad \lambda_* = -4.51, \quad (4.15)$$

apart from the Gaussian fixed point, which is always a solution. There is an additional solution, with $g_* < 0$, but since gravity is universally attractive, we disregard this possibility. The flow of the beta functions (4.12) and (4.13) is displayed in the phase diagram in Fig. 4.2 for the matter content of the SM. In the following, we shall use the general parameterizations (4.10) and (4.6) as input for our generalized analysis, and we will only use the specific values (4.15) when reproducing the results of [9].

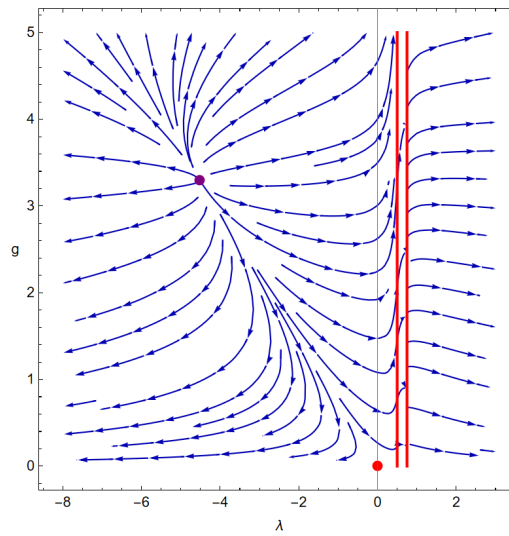


Figure 4.2: RG flow for the gravitational sector when we account for the matter content of the SM. It features a fully UV-attractive interacting fixed point and a saddle point at the origin. The vertical red lines represent divergences in the beta-functions and no trajectory can cross those lines.

4.3 Matter couplings and their beta functions

For the matter sector of the SM, we will focus on two types of couplings, which are the three gauge couplings and the Yukawa couplings for the top and bottom quarks. This choice is for simplicity, and because these are the only ones that are essential for our study. The running of Yukawa couplings, from which we can extract the masses of the quarks, is affected by the gauge couplings, but not vice versa. This will be reflected in their beta functions and their dependence on the different couplings.

First, we present the beta function for the gauge couplings, which have the following structure [9],

$$\beta_{g_i} = c_i \frac{g_i^3}{16\pi^2} - g_i \cdot g f_g(\lambda), \quad (4.16)$$

where $i = 1, 2, 3$ for the three gauge groups. The numerical factors are $c_1 = 41/10$, $c_2 = -19/6$, $c_3 = -7$. We distinguish two terms: the first one comes from the SM one-loop contribution, while the second one arises from the gravitational contribution. The numerical factors are negative for the SU(2) and SU(3) couplings, which are already asymptotically free in the SM, while the contribution for the U(1) gauge coupling is positive and features a Landau pole in the absence of gravity. It is only thanks to the addition of gravity that this coupling can instead become asymptotically free as well as the other two. The gravitational contribution is equal for all of them, and in the Einstein-Hilbert truncation are given by

$$f_g(\lambda) = \frac{5(1 - 4\lambda)}{18\pi(1 - 2\lambda)^2}. \quad (4.17)$$

Regarding the Yukawa sector, their beta functions [9] are presented as follows

$$\beta_{y_t} = \frac{1}{32\pi^2} \left(9y_t^3 + 3y_b^2 y_t + y_t \left(-16g_3^2 - \frac{9}{2}g_2^2 - \frac{17}{10}g_1^2 \right) \right) + y_t \cdot g f_y(\lambda), \quad (4.18)$$

$$\beta_{y_b} = \frac{1}{32\pi^2} \left(9y_b^3 + 3y_t^2 y_b + y_b \left(-16g_3^2 - \frac{9}{2}g_2^2 - \frac{1}{2}g_1^2 \right) \right) + y_b \cdot g f_y(\lambda). \quad (4.19)$$

With a gravitational contribution

$$f_y(\lambda) = \frac{96 + \lambda(-235 + \lambda(103 + 56\lambda))}{12\pi(3 + 2\lambda(-5 + 4\lambda))^2}. \quad (4.20)$$

We see that they are similar to each other, just exchanging $y_t \leftrightarrow y_b$ and with a different contribution from the g_1 coupling, since both quarks have different hypercharges. It is worth saying that here, we are using the normalized U(1) group coupling g_1 , which is related to the hypercharge coupling used in the SM by $g_1 = \sqrt{\frac{5}{3}}g_Y$.

4.4 Fixed points of gravity-matter systems and their stability

The first task to draw predictions from ASQG is to find the fixed points of our gravity-matter system. In order to do this, in preparation for the main goal of this thesis, we will take the gravitational fixed point values, g_* and λ_* , as external parameters that we

do not specify yet. Once we have studied the fixed points in our system, the next step is to analyze their stability to determine the possible trajectories that can be safe. To do so, we will need to compute the stability matrix,

$$(\mathcal{S})_{ij} = \frac{\partial \beta_{\alpha_i}}{\partial \alpha_j}, \quad (4.21)$$

where α_i are the couplings in our system, i.e. g , λ , g_1 , g_2 , g_3 , y_t and y_b . The matrix needs to be evaluated at the fixed point we are considering. From it, its eigenvalues are going to be the critical exponents with a minus sign, and the eigenvectors are going to define the corresponding relevant and irrelevant directions.

First, let us take care of the gauge couplings only. Apart from the Gaussian fixed point, which is the trivial solution, we can find other interacting fixed points by solving $\beta_{g_i} = 0$. Since each beta function is only dependent on its respective coupling, the solutions are

$$g_i^* = \frac{4\pi}{\sqrt{c_i}} \sqrt{g_* f_g(\lambda_*)}, \quad (4.22)$$

which will only be real when $c_i^{-1} g_* f_g(\lambda_*) \geq 0$. The fixed points that we found are displayed in the table 4.1.

g_1^*	g_2^*	g_3^*	Condition
0	0	0	Always exists
$\frac{20}{3} \sqrt{\frac{\pi}{41}} \frac{\sqrt{g_*(1-4\lambda_*)}}{ 1-2\lambda_* }$	0	0	$g_* > 0$ & $\lambda_* < \frac{1}{4}$
0	$4 \sqrt{\frac{5\pi}{57}} \frac{\sqrt{g_*(4\lambda_*-1)}}{ 1-2\lambda_* }$	0	$g_* > 0$ & $\lambda_* > \frac{1}{4}$
0	0	$\frac{2}{3} \sqrt{\frac{10\pi}{7}} \frac{\sqrt{g_*(4\lambda_*-1)}}{ 1-2\lambda_* }$	$g_* > 0$ & $\lambda_* > \frac{1}{4}$
0	$4 \sqrt{\frac{5\pi}{57}} \frac{\sqrt{g_*(4\lambda_*-1)}}{ 1-2\lambda_* }$	$\frac{2}{3} \sqrt{\frac{10\pi}{7}} \frac{\sqrt{g_*(4\lambda_*-1)}}{ 1-2\lambda_* }$	$g_* > 0$ & $\lambda_* > \frac{1}{4}$

Table 4.1: Fixed point values for the gauge couplings depending on the parameters g_* and λ_* , being the gravitational couplings fixed point values.

The Gaussian fixed point exists for every value of g_* and λ_* , and will render the gauge sector asymptotically free. There are other four fixed points for which at least one of the couplings is non-zero. Since each equation is independent of the rest, every combination of solutions should be possible, so if there is an interacting and a non-interacting solution for each of the couplings, we should find in principle $2^3 = 8$

solutions for the system of equations. However, if we look at the first term of the beta functions, we see that the g_1 coupling has opposite sign to the other couplings. Therefore, its non-zero value is not compatible with a non-zero value for the other two couplings, as one requires $\lambda_* < 1/4$, while the others require $\lambda_* > 1/4$ for the solution to be real. This limits the number of solutions to just five.

Now that we have the fixed points, we study their stability. To do so, we construct a stability matrix restricted to the space of gauge couplings. The fact that the beta functions are only dependent on their respective coupling makes things simpler, because the matrix is going to be diagonal. As a consequence, the entries are just its eigenvalues, or if we reverse the sign, then they will be the critical exponents. In fact, they are easily computed as

$$\theta_{g_i} = - \left. \frac{\partial \beta_{g_i}}{\partial g_i} \right|_{g_i^*} = -c_i \frac{3g_i^{*2}}{16\pi^2} + g_* f_g(\lambda_*) = \begin{cases} g_* f_g(\lambda_*) & \text{if } g_i^* = 0 \\ -2g_* f_g(\lambda_*) & \text{if } g_i^* \neq 0. \end{cases} \quad (4.23)$$

Now, if we consider $g_* > 0$ as the only physical possibility, then the stability depends just on the value of λ_* . The critical exponent is positive for $g_i^* = 0$ if $\lambda_* < 1/4$ and for $g_i \neq 0$ if $\lambda_* > 1/4$. So if we want to avoid any divergence in the running of these couplings, i.e. a Landau pole, then we need to choose a different region for λ_* depending if the value of the coupling at the fixed point is zero or not. Nonetheless, since the two regions do not overlap, this means that in order to achieve this all the couplings need to be either interacting or non-interacting at the same time. Since there is no fixed point in which $g_i^* \neq 0$ for all of the three couplings, the only remaining possibility is the Gaussian fixed point. This means that $g_1^* = g_2^* = g_3^* = 0$, i.e. when all couplings are asymptotically free, is the only fixed point that does not feature any divergence in the UV. This is true only provided $\lambda_* < 1/4$. Any other fixed point will feature a Landau pole in at least one of the couplings for every value of λ_* in which the fixed point exists.

We now do the same analysis for the Yukawa sector. When we pick one of the five fixed points for the gauge couplings, we can then find the solutions for the Yukawa fixed points, by solving the equations $\beta_{y_t} = 0$ and $\beta_{y_b} = 0$. For every choice of fixed point values for the gauge couplings, we can always find four different fixed points for the Yukawa couplings, being one of them $y_t^* = y_b^* = 0$. This makes a total of $5 \times 4 = 20$ different fixed point solutions for the matter sector. We have already discussed that the gauge sector needs to be asymptotically free if we want to have a UV-completion, so we are only left with four possible solutions for the Yukawa couplings. We display in

table 4.2 the Yukawa fixed points in the case when we set all the gauge couplings to zero.

y_t^*	y_b^*	Condition
0	0	Always exists
$\frac{4\pi\sqrt{2}}{3}\sqrt{-g_*f_y(\lambda_*)}$	0	$g_* > 0$ & $\lambda_* < -3.28$
0	$\frac{4\pi\sqrt{2}}{3}\sqrt{-g_*f_y(\lambda_*)}$	$g_* > 0$ & $\lambda_* < -3.28$
$\frac{2\pi\sqrt{6}}{3}\sqrt{-g_*f_y(\lambda_*)}$	$\frac{2\pi\sqrt{6}}{3}\sqrt{-g_*f_y(\lambda_*)}$	$g_* > 0$ & $\lambda_* < -3.28$

Table 4.2: Fixed point values for the Yukawa couplings depending on the parameters g_* and λ_* , being the gravitational couplings fixed point values. These are the fixed points that are compatible with a Gaussian fixed point for the gauge sector.

In the case where $g_i^* \neq 0$, we can also obtain analytic expressions for the fixed point values. The fixed points lying on the axes are

$$y_t^* = \frac{1}{3}\sqrt{16g_3^{*2} + \frac{9}{2}g_2^{*2} + \frac{17}{10}g_1^{*2} - 32\pi^2g_*f_y(\lambda_*)} , \quad y_b^* = 0 , \quad (4.24)$$

and

$$y_t^* = 0 , \quad y_b^* = \frac{1}{3}\sqrt{16g_3^{*2} + \frac{9}{2}g_2^{*2} + \frac{1}{2}g_1^{*2} - 32\pi^2g_*f_y(\lambda_*)} . \quad (4.25)$$

In this case, they will not be at the same distance from the origin as long as $g_1^* \neq 0$. For the fixed point in which both couplings are non-zero, their values are

$$y_t^* = \frac{1}{2}\sqrt{\frac{16}{3}g_3^{*2} + \frac{3}{2}g_2^{*2} + \frac{23}{30}g_1^{*2} - \frac{32}{3}\pi^2g_*f_y(\lambda_*)} ,$$

$$y_b^* = \frac{1}{2}\sqrt{\frac{16}{3}g_3^{*2} + \frac{3}{2}g_2^{*2} - \frac{1}{30}g_1^{*2} - \frac{32}{3}\pi^2g_*f_y(\lambda_*)} . \quad (4.26)$$

These again, will not satisfy $y_t^* = y_b^*$ if $g_1^* \neq 0$, but rather the relation

$$y_t^{*2} - y_b^{*2} = \frac{1}{5}g_1^{*2} . \quad (4.27)$$

When we set all the gauge couplings to zero in the Yukawa beta functions, we can see that one function turns into the other when we exchange y_t and y_b . In this case, the fixed points will show a symmetry in the values of the couplings, see Fig. 4.3. There is a fixed point in which both couplings have the same value, and another two in which

the values are exchanged, being one of them zero and the other non-zero. This is always the case as long as we set $g_1^* = 0$, otherwise this symmetry is broken due to the fact that, as we mentioned, both quarks have different hypercharges.

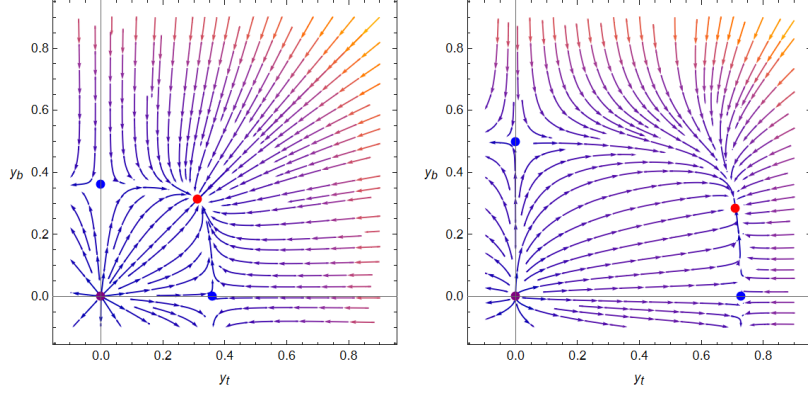


Figure 4.3: Projection of the RG flow on the space of the Yukawa couplings and its four fixed points, when setting $g_* = 3.29$, $\lambda_* = -4.51$ and $g_2^* = g_3^* = 0$. The fixed point value of the gauge coupling g_1 is set to zero on the left panel and non-zero in the right panel.

Apart from the Gaussian fixed point, the other interacting points are only real when $\lambda_* < -3.28$, being compatible at the same time with the requirement $\lambda_* < 1/4$ from the gauge sector. This happens again due to the presence of a square root in their analytical expression. Assuming $g_* > 0$, we need to require then that $f_y(\lambda_*) < 0$. We can see this is the case for $\lambda_* < -3.28$, while for the rest of values the function is positive. The other region of validity would correspond to $g_* < 0$, but we disregard this possibility due to the fact that a negative Newton coupling would imply repulsive gravity, as opposed to observations. Furthermore, since its beta function does not allow it to change sign, then the Newton coupling must be positive at all scales. Thus, the only viable region remaining is $g_* > 0$ and $\lambda_* < -3.28$, which satisfies $f_y(\lambda_*) < 0$. Indeed, because the SM contribution for the Yukawa couplings, neglecting the effect of gauge couplings, is positive; then we need a negative gravitational contribution to balance the flow and be able to reach a fixed point regime.

With these four fixed points, we can insert them into a stability matrix restricted to the Yukawa couplings only. This will give us an idea of how the couplings behave close to the respective fixed points. For a qualitative picture, it suffices to take a look at Fig. 4.3. If we perform the calculations, we will see that, in the case $g_i^* = 0$ for all gauge couplings we have:

- For the Gaussian fixed point, we have two relevant directions with critical exponents $-g_* f_y(\lambda_*)$. This is the case where all the matter couplings are asymptotically free and the Gaussian fixed point is fully UV-attractive. In the left panel of Fig. 4.3, it is marked as a purple dot.
- For the fixed points that lie on the axes, marked as blue dots in the left panel of Fig. 4.3, we have saddle points with eigenvectors aligned with the axes. The relevant direction corresponds to the coupling whose fixed point value is zero and has a critical exponent of $-\frac{2}{3}g_* f_y(\lambda_*)$. On the other hand, the coupling with a non-zero fixed point value has a negative critical exponent, with value $2g_* f_y(\lambda_*)$. So, we have an asymptotically free coupling and another with an interacting fixed point that is UV-repulsive.
- For the fixed point such that $y_t^* = y_b^* \neq 0$, we have two irrelevant directions that are not aligned with the axes and have critical exponents $g_* f_y(\lambda_*)$ and $2g_* f_y(\lambda_*)$. This point is fully UV-repulsive and is marked in the left panel of Fig. 4.3 as a red dot.

We see that some fixed points feature irrelevant directions. These directions correspond to parameters that are not free, meaning that if we want to avoid divergences, then their values at some RG scale need to be carefully fixed. It is precisely the existence of irrelevant directions and the condition of having a UV-completion that allow us to predict parameters.

Now the issue is selecting initial conditions that do not lead to divergences but also that reproduce reasonably well the results we expect in the IR. A problem we have after discussing the fixed points in the Yukawa sector is that if we fix a coupling to zero, then it will not run at all and will be fixed there for all scales. So obviously we cannot set any initial condition to be $y_{t,b}(k_0) = 0$ for any RG scale k_0 . But then, the only possibility to prevent the flow from going into an irrelevant direction is if another relevant direction is a linear combination of different couplings. It turns out that when we take the gravitational couplings into account, we get relevant directions that are linear combinations of g , λ , and y_t or y_b (depending on the fixed point). And thanks to this fact, asymptotically safe trajectories exist.

4.5 Asymptotically safe trajectory

After analyzing the existence of fixed points and whether these are stable or not, it is time to numerically solve the set of differential equations $k\partial_k\alpha_i = \beta_{\alpha_i}$ to find the RG trajectories for the couplings.

As we have seen, there are four possible fixed points towards which the Yukawa couplings can flow. However, the Gaussian fixed point is completely UV-attractive, and thus no prediction is drawn from it. We also have an interacting fixed point for both y_t and y_b , which is fully IR-attractive, and therefore we disregard it as no trajectories will approach it. The interesting fixed points are then the ones in which one of the couplings is asymptotically free, because, as we will see, it is based on those for which we can predict the masses of the quarks. A priori any of the two masses m_t and m_b , i.e. the masses of the top and bottom quarks respectively, can be predicted in this scenario. However, based on the experimental fact that m_t is considerably larger than m_b , the mass that we can reasonably predict is actually the one of the top quark. In fact, it is not really a prediction but a post-diction, because it will only be given after we take as an input the mass of the bottom quark as well as the values of the gauge couplings, depending strongly on these.

It is also worth mentioning that a difference in mass for both quarks is dynamically generated through the RG flow, as argued in [122]. If we consider the ratio y_t/y_b , and compute its beta function,

$$\beta_{\frac{y_t}{y_b}} = \partial_t \left(\frac{y_t}{y_b} \right) = \frac{\partial_t y_t \cdot y_b - y_t \cdot \partial_t y_b}{y_b^2} = \frac{1}{32\pi^2} \frac{y_t}{y_b} \left[6(y_t^2 - y_b^2) - \frac{12}{10} g_1^2 \right] \quad (4.28)$$

If the ratio $y_t/y_b \leq 1$, meaning that $y_b \geq y_t$, then this beta function becomes negative, and consequently, the ratio will grow when we flow towards the IR. Even if the ratio approaches 1 from above, the same result holds; that is, if we have $y_t^* > y_b^*$, that relation will continue to be true in the IR, in agreement with experimental data.

We can also analyze the behavior of the beta functions for y_t and y_b . For perturbative values of the couplings, i.e. smaller than one, the leading order term will be given by the one with the lowest power. In this case, they are the linear terms in the Yukawa couplings. If we neglect the contribution of gravity, assuming we are in the IR and the gravitational couplings are negligible, we see that the linear term dominating the evolution is negative, since it is multiplied by a negative factor that contains the

gauge couplings squared. This dominant negative contribution will make the couplings decrease when we flow towards the UV. As a result, the IR values of the couplings need to be larger than their fixed point values. By plugging $g_* = -3.29$ and $\lambda_* = -4.51$ as representative values for the parameters, we see that the fixed point value for the bottom Yukawa coupling y_b^* when $y_t^* = 0$ corresponds to a higher value than the expected value in the IR according to experiments, eliminating the possibility to match the solution with experimental data. This does not happen with the fixed point value for y_t , making it possible to find a solution compatible with the experimentally determined top quark's mass. Due to this reasoning, we can believe that the fixed point where $y_t^* \neq 0$ and $y_b^* = 0$, is the one that will be relevant for our purpose.

Finally, we can construct the full stability matrix containing all our couplings, including matter and gravitational ones. As we know, evaluating this matrix at a fixed point will allow us to scrutinize whether we can have a UV-completion and when it is possible to make predictions. If we analyze what happens with the fixed point that we were discussing previously, the one with $g_1^* = g_2^* = g_3^* = y_b^* = 0$ and $y_t^* \neq 0$ we will find a UV-completion that gives us a prediction for the top quark's mass. To serve as an example, let us proceed the computation using the fixed point values for the gravitational couplings that we found for the matter content in the SM, i.e. $g_* = 3.29$, $\lambda_* = -4.51$, as done in [9]. In the next chapter, we will study what happens for generic values. The numerical results are:

- In the gauge sector, all three couplings define relevant directions with a critical exponent of 5.52×10^{-2} .
- For the Yukawa bottom coupling, it also defines a relevant direction with a critical exponent of 2.48×10^{-3} .
- The Yukawa top coupling defines an irrelevant direction, with a critical exponent of -7.43×10^{-3} .
- There are two relevant directions which are linear combinations of the gravitational couplings and the Yukawa top coupling:
 - One with critical exponent 4 that is a linear combination of y_t and λ .
 - Another with critical exponent 2 that is a linear combination of y_t , g and λ .

So, we can plug in as initial conditions for the free parameters: $g_1(k_{\text{IR}}) = \sqrt{\frac{5}{3}}0.3583$, $g_2(k_{\text{IR}}) = 0.64779$, $g_3(k_{\text{IR}}) = 1.1666$ [125], $y_b(m_b) = \sqrt{2}m_b/v$ with $k_{\text{IR}} = 173$ GeV, $m_b = 4.18$ GeV [137] and $v = 246$ GeV. On the other hand, for y_t we will set an initial condition in the far UV, at an arbitrary scale $k_{\text{UV}} \gg M_{\text{Pl}}$. The idea behind this, is to select a trajectory that is aligned as closed as possible with a relevant direction, leading the flow towards the fixed point. To do this, we pick the initial condition arbitrarily close to the fixed point, with a precise value given by the components of the relevant direction and the known value of the cosmological constant at the UV scale, see Fig. 4.4 for a visualization of this idea.

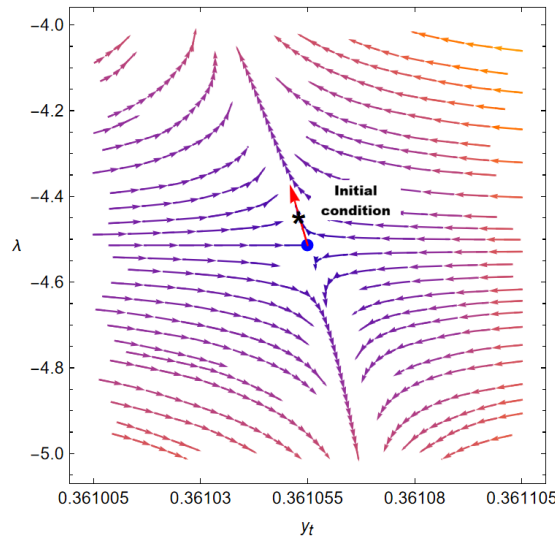


Figure 4.4: RG flow projected in the y_t - λ plane at the fixed point for the rest of couplings. We can see a relevant direction which is a linear combination of both couplings. The initial condition is fixed to lie precisely in that direction and to be very close to the fixed point. This is a depiction of the idea, it does not show the actual scale.

In this way, the coupling y_t will be asymptotically safe, approaching its fixed point value while all the other couplings approach zero. However, due to the existence of an irrelevant direction, this asymptotically safe trajectory is highly unstable, and any small deviation will drive the coupling to either zero or infinity. Since we cannot have infinite precision using numerical methods, we can trust our solution only up to arbitrarily high scales, but not until infinite RG scales. This will not be a problem, because the approximation is robust enough to assume we are actually in an asymptotically safe regime if we follow the previous indications. The procedure that has just been detailed serves as a general way to construct asymptotically safe trajectories, and therefore can be employed to retrodict the top mass regardless of the gravitational parameters. This section just provides a particular example, that can be contrasted with the results of [9], but could be applied to other scenarios.

Given these details, we can proceed to solve the differential equations numerically to get the trajectories for every coupling. The results obtained for the gravitational parameters and initial conditions previously mentioned in this section are displayed in Fig. 4.5.

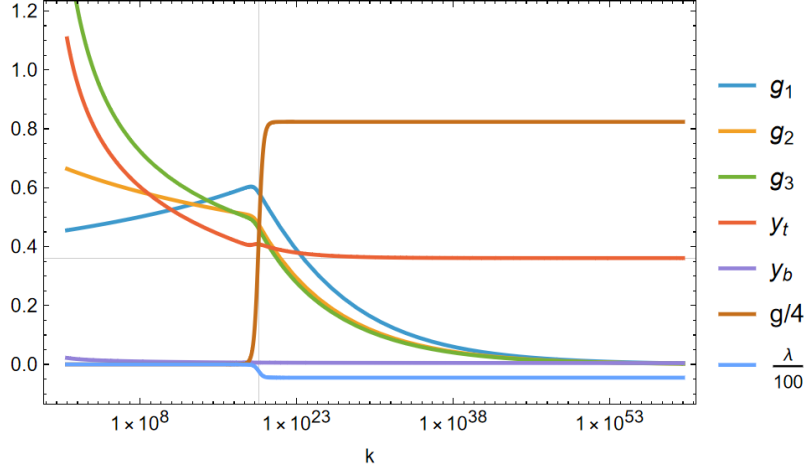


Figure 4.5: Running of all couplings for $g_* = 3.29$ and $\lambda_* = -4.51$. The Yukawa coupling y_t is asymptotically safe while the rest of the SM couplings are asymptotically free.

4.6 Retrodiction of the top mass

To extract the value for the top mass, we simply evaluate the solution at the experimental scale, $m_t = y_t(k_{\text{IR}})v/\sqrt{2}$. For the case $g_* = 3.29$, $\lambda_* = -4.51$, we obtain the value $m_t = 161.03$ GeV, in excellent agreement with the result in [9]. However, this is not the physical mass, but a *running* mass. The physical mass corresponds to the pole in the fermion propagator, and it is the mass that we actually measure in experiments. To obtain the physical mass from the running mass we need to include QCD loop corrections. If we perform the calculations, according to [138], we can see that the corresponding pole mass that we get in this case is $M_{t,\text{pole}} = 170.53$ GeV, which is remarkably close to the experimentally determined mass [126], $M_{t,\text{pole}} = (172.95 \pm 0.53)$ GeV, especially given that FRG calculations come with a systematic error due to truncations.

This result was however obtained by fixing $g_* = 3.29$, i.e., fixing the quantum gravity scale $k_{\text{FP}} \simeq M_{\text{Pl}}$. In a scenario in which ASQG acts as a bridge between EFT and a more fundamental theory, such as string theory [35, 73, 74, 139], in order to still make use of the top-mass prediction from ASQG one would need $k_{\text{FP}} < k_{\text{QFT}}$. If, in particular,

the fundamental theory is string theory, then $k_{\text{QFT}} = M_s \ll M_{\text{Pl}}$. The natural question to ask is then, how is the top-mass post-diction affected by k_{FP} ? As k_{FP} is fixed by g_* , answering this question boils down to studying the whole system as a function of the gravitational fixed-point values.

In the following chapter, we will study how the change of the parameters g_* and λ_* modifies the result of the top mass. As we mentioned, these parameters that we can freely vary are dependent on the number of matter fields. Therefore, it is also interesting to study how they change when we select another set of (N_S, N_V, N_W) .

The scale(s) of Quantum Gravity

In this chapter we will discuss how the post-diction on the top mass is affected by the quantum gravity scale, which we will need to define first. This is the scale at which gravitational fluctuations take over and gravity becomes strongly interacting, and it is usually regarded as being fixed as the Planck scale. However, this notion in asymptotic safety is modified to account for a scale in which we transition from a classical weakly-coupled gravity to the fixed point regime. The main idea is to explore the space of gravitational parameters g_* and λ_* and see how the top mass changes, delimiting a region in which it is possible to have a post-diction that is compatible with experimental data within a UV-complete theory.

5.1 Top-mass constraints on the quantum gravity scale

The original computation of the top mass in [9], which we have reproduced in the last section of the previous chapter, took $g_* \sim \mathcal{O}(1)$, corresponding to a fixed-point scale $k_{\text{FP}} \sim M_{Pl}$. However, as previously explained, in a scenario of effective ASQG, the compatibility with $k_{\text{QFT}} < M_{Pl}$ requires lowering the fixed point scale to $k_{\text{FP}} < k_{\text{QFT}}$. Based on the relation (4.9), this can be done by changing g_* . However, this may also impact the compatibility with experimental data. Reverse-engineering the logic, in this section we use the experimental constraints on the top mass to put a constraint on the fixed point scale k_{FP} .

The first step to reach our goal is to understand the behavior of the solution $y_t(k)$, particularly how the running in the UV is altered by the gravitational parameters. For that, two main things will determine the trajectory: the fixed point value and the initial condition.

The fixed point value for y_t can be obtained analytically, as we can see in table 4.2. It comes from solving $\beta_{y_t} = 0$ by setting all the gauge couplings and the bottom Yukawa coupling to zero, i.e. $g_i^* = y_b^* = 0$, yielding

$$y_t^* = \sqrt{-\frac{32\pi^2}{9} g_* f_y(\lambda_*)}, \quad (5.1)$$

which exists for $\lambda_* < -3.28$. This solution is displayed in Fig. 5.1 for different values of g_* and λ_* .

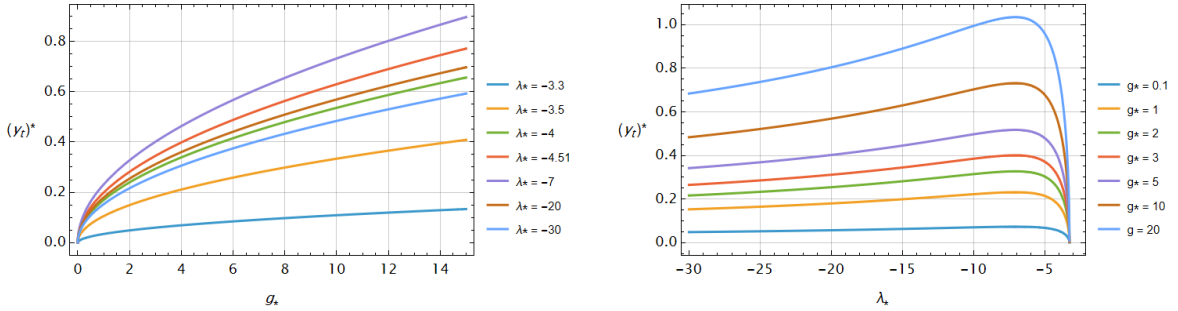


Figure 5.1: Fixed point value for the top Yukawa coupling as a function of g_* (left panel) and λ_* (right panel).

As we can see from expression (5.1), the fixed point value scales as $y_t^* \sim \sqrt{g_*} = k_{\text{FP}}/M_{\text{Pl}}$ for a fixed value of λ_* . Thus, the fixed point value increases proportionally to the quantum gravity scale k_{FP} . The rate of increase is given by the value of the cosmological constant, as a factor $\sqrt{-f_y(\lambda_*)}$.

The algorithm we used in the previous chapter to retrodict the top mass was based on finding an asymptotically safe trajectory. This trajectory is unique for a certain fixed point value for the top Yukawa coupling. This means that for any combination of gravitational parameters that yield the same value for y_t^* , the post-diction for the top mass will be the same. Therefore, there is a map from $y_t^*(g_*, \lambda_*)$ to the running top mass predicted by asymptotic safety. The new challenge is finding this relation, because translating a point in (g_*, λ_*) to a fixed point is easy, just plugging it in the expression (5.1), but finding the running top mass in the IR requires solving differential equations numerically.

First thing to do is finding numerically the fixed point value y_t^* associated to each value of the top mass. For that, we implement an algorithm that numerically searches for the corresponding value of y_t^* given m_t and a value for λ_* . The fixed point value of the

cosmological constant that we use is irrelevant, just makes it easier to look for y_t , since we are just varying and checking the asymptotic safety prediction until we find the desired mass. Once this is done, we can represent in a plot the lines in the parameter space (g_*, λ_*) that correspond to a particular prediction coming from an asymptotically safe trajectory. The result of this process is represented in Fig. 5.2, constituting the first main result of this section.

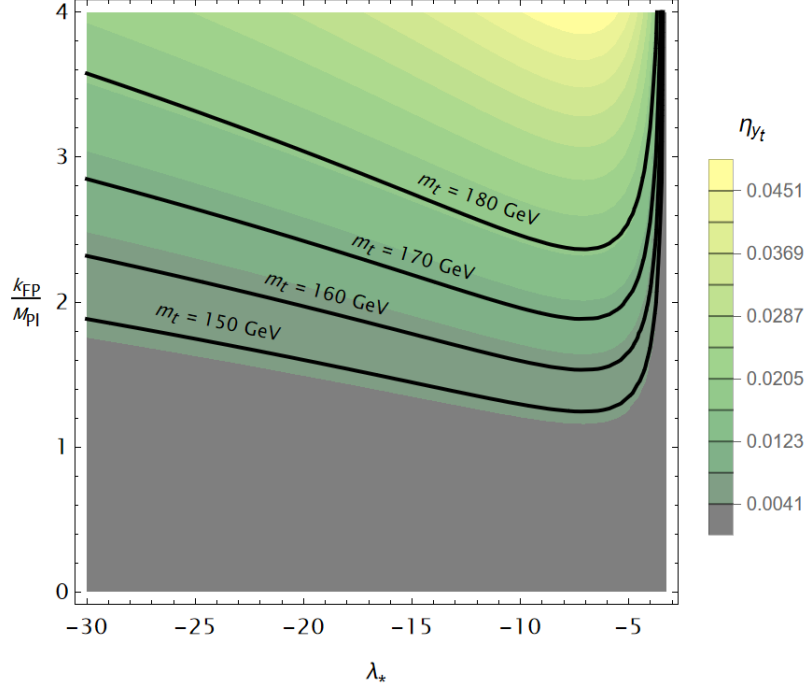


Figure 5.2: Different running top masses predicted by an asymptotically safe trajectory depending on the scale of the fixed point, k_{FP} , and the cosmological constant fixed point value, λ_* . The shaded region in the background represents the magnitude of the anomalous dimension associated to the top Yukawa coupling.

The lines of equal mass in Fig. 5.2 correspond to lines of equal y_t^* for which such mass is retrodicted. We also include a shaded background indicating the value of the anomalous dimension associated to the top Yukawa coupling. As we can see, lower values of the top mass also correspond to lower anomalous dimension. This is also an indicator of the “perturbativity” of such coupling, since η_{y_t} is computed here as the canonical dimension minus the critical exponent, or equivalently,

$$\theta_{y_t} = [y_t] - \eta_{y_t}. \quad (5.2)$$

In other words, η_{y_t} is just measuring how much we deviate from canonical scaling. Given that the mass dimension of the Yukawa couplings is zero, then the anomalous dimension is just the critical exponent with a minus sign. This critical exponent is

computed numerically from the same stability matrix that we constructed for the numerical solution of $y_t(k)$, in chapter 4.

Now, coming back to the conjecture we discussed at the end of the last chapter, we can find what sets of gravitational parameters can predict a top mass that is compatible with experimental data. This will require two steps:

- First, finding the running masses that correspond to the experimental bounds of the top pole mass. For that, we implement an algorithm that, based on the results by [138], can invert the relation between the running mass and the pole mass. Then we can apply it to find what running masses we are looking for.
- Next step is to, again, look for the fixed point values y_t that lead to the trajectories predicting the bounds on the running mass that we found in the previous step. This way, we can find the region in the parameter space that gives values of the top mass that are compatible with data.

By following this procedure, we find that if we want to stay within a region that predicts a pole mass of

$$172.42 \text{ GeV} < M_{t,pole} < 173.48 \text{ GeV} , \quad (5.3)$$

then the running mass must stay between the following values,

$$162.84 \text{ GeV} < m_t < 163.85 \text{ GeV} . \quad (5.4)$$

This translates to a region of fixed point values of

$$0.3754 < y_t^* < 0.3832 . \quad (5.5)$$

These results indicate the region in which the compatible post-diction comes from an asymptotically safe trajectory. However, this is a strong requirement, since requiring compatibility with experiments does not necessarily imply that the trajectory must be safe at all. Imposing different initial conditions at an IR scale, e.g. $k_{\text{IR}} = 173 \text{ GeV}$, can give rise to three different scenarios, all compatible with the observed value of the top mass: the trajectory can be asymptotically free, asymptotically safe, or unsafe (leading to a divergence at a Landau pole). This is shown in Fig. 5.3. Once we pick $y_t(k_{\text{IR}})$ to be at the experimental value, the type of trajectory will be determined by the gravitational parameters. In Fig. 5.3, we represent with a solid line the trajectory that

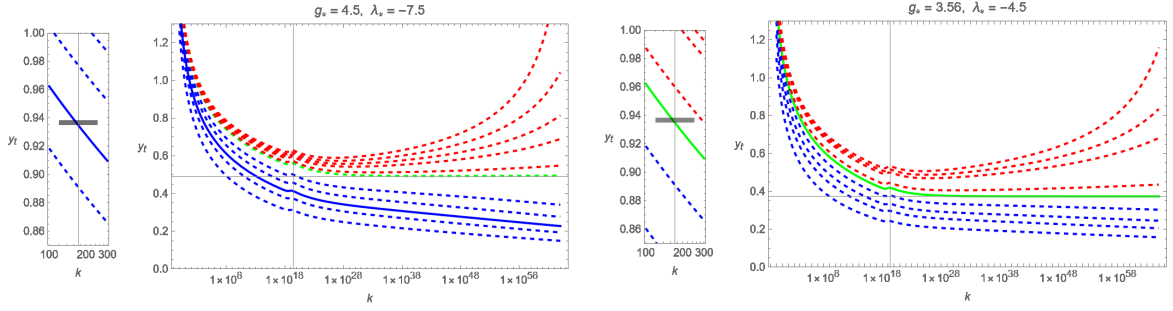
is compatible with experiments, and we can see that changing the values of g_* and λ_* leads to a different scenario. The dotted lines in the same figure correspond to other initial conditions outside of experimental bounds.

The physical interpretation of these scenarios depends on the ordering of the relevant scales discussed in Sect. 4.1. If the QFT description remains valid above the fixed-point scale, k_{FP} , the fixed point acts as attractor controlling the UV behavior of the couplings. On the other hand, if a more fundamental theory takes over below the fixed point scale, $k_{\text{QFT}} < k_{\text{FP}}$, the interpretation changes significantly. In this case, the QFT description breaks down before reaching the fixed point regime and we excite new degrees of freedom that drive the flow of the couplings. The fact that the top Yukawa coupling followed the same trajectory as the one predicted by ASQG would appear as a nontrivial coincidence, since the fixed point would no longer be responsible for attracting the flow.

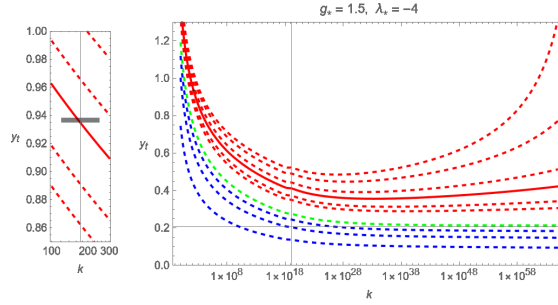
The answer to why the gravitational parameters change the behavior of the trajectory is simple, and it is because they affect the fixed point value for the top Yukawa coupling. As a result, for the same initial condition in the IR, a lower y_t^* makes the trajectory grow indefinitely, moving away from the fixed point and eventually diverging. This trajectories are unsafe and feature Landau poles, whose position can be calculated, but that will be the task in the next section. On the other hand, if we increase the value y_t^* without modifying the initial condition, the opposite behavior will take place, and the $y_t(k)$ will decrease driven by the anti-screening gravitational contribution. In that case, the top Yukawa coupling is not able to reach such a high fixed point value, and instead flows towards the GFP, i.e. becomes asymptotically free.

We discussed that a UV-completion can be realized by any fixed point, be it safe or free, as long as it can drive the RG flow towards it. Because of this, we can still UV-complete the theory with the free fixed point, instead of the safe one. This means that a certain value of the top mass in the IR can not only come from an asymptotically safe trajectory, but also from an asymptotically free one. This is much less restrictive, because it allows for more trajectories to be considered compatible and UV-complete.

In particular, if we keep fixed the initial condition at a particular value, there is a unique line in the space of gravitational parameters which corresponds to a safe trajectory. This is because, as we explained in the last chapter, safe trajectories are highly unstable. Now, if we change the gravitational parameters, the fixed point y_t^* will change consequently. If it increases, we argued that the trajectory will become free,



(a) Compatible prediction in an asymptotically free trajectory. (b) Compatible prediction in an asymptotically safe trajectory.



(c) Compatible prediction in an unsafe trajectory.

Figure 5.3: A prediction compatible with experimental data (solid line) can come from three different scenarios: a safe (green line), free (blue lines) or unsafe (red lines) trajectory, depending on the values of the gravitational parameters. Dotted lines represent trajectories that are not compatible with the experimentally determined top pole mass. Compatible trajectories must pass by the gray rectangle in the left panel of each figure, which delimits the experimental bounds in expression (5.4).

whereas if it decreases, the trajectory becomes unsafe. Knowing this, together with the fact that the fixed point value (5.1) increases proportionally to k_{FP} , we conclude that the lines of constant mass in Fig. 5.2 are actually boundary lines. This means that any point above those lines corresponds to the prediction of the same mass, but for a free trajectory. And any point below it can also correspond to the same prediction, but in an unsafe trajectory.

This way, we plot in Fig. 5.4 the region in the parameter space in which we can retrodict a top mass compatible with experiments within a UV-completion. The boundary in that figure corresponds to the lower bound on the running top mass, i.e. the lower bound in (5.4), emerging from an asymptotically safe trajectory. Any point above the blue line correspond to the same IR value of the top Yukawa coupling, or even above it with an increasing range as we move upwards, coming from an asymptotically free trajectory. The region outside this bound can also retrodict a top mass compatible with experiments, however, only for unsafe trajectories.

We conclude that there is a minimum scale k_{FP} for which we have compatibility with data within a UV-completion. By looking for this minimum at the boundary region in the parameter space, we can obtain this scale we are referring to. The way to compute this analytically is by, first, finding the minimum of y_t^* for λ_* , which amounts to finding the minimum of $f_y(\lambda_*)$. This is obtain by solving

$$\left. \frac{\partial f_y(\lambda_*)}{\partial \lambda_*} \right|_{\lambda_*=\lambda_{*,\min}} = 0 \quad (5.6)$$

Once we obtain $\lambda_{*,\min}$, we solve for $g_{*,\min}$ in the expression for y_t^* when this value is equal to the lower bound compatible with experimental data, i.e. the lower bound on (5.5). This way, we obtain that the minimum occurs at $k_{\text{FP}} \simeq 1.62 M_{\text{Pl}}$, as we can see in Fig. 5.4.

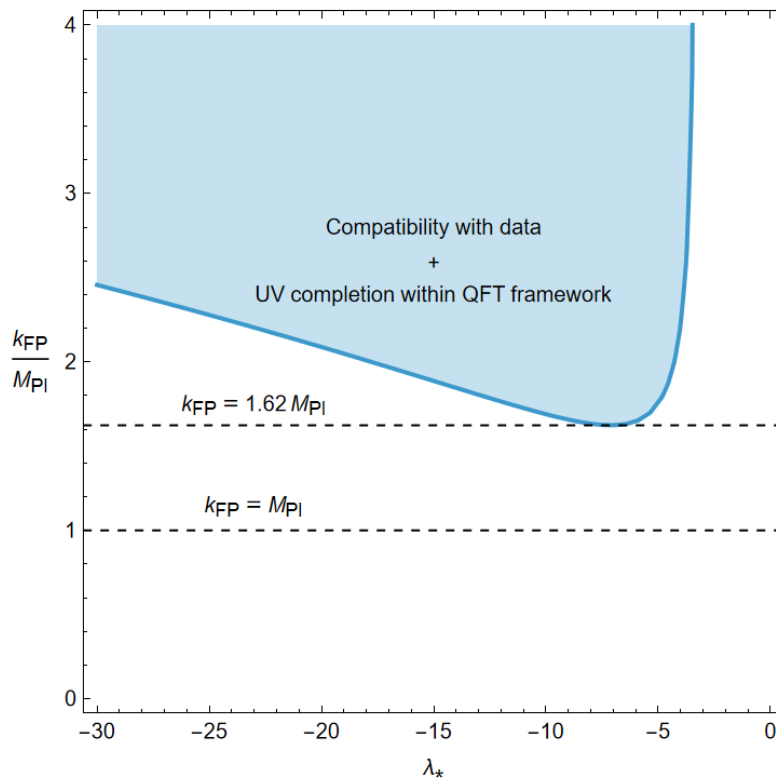


Figure 5.4: Region in the space of k_{FP} and λ_* in which we can retrodict a top mass compatible with experiments from a UV-complete theory of the SM plus gravity. The boundary of the region corresponds to safe trajectories with an IR value of the top Yukawa coupling corresponding to the lower experimental bound on the top mass.

This result is the first constraint we find on the fixed point scale, just by requiring the compatibility of asymptotic safety with the observed top mass and assuming a UV-complete theory. Other analysis may provide different results, and better approxi-

mations can refine the precise value. However, if we found other results that overlap with this, further constraints might be imposed.

5.2 Landau poles

In this section, we turn our attention to the unsafe trajectories that feature a Landau pole, diverging at a finite RG scale. The presence of such a pole signals a necessary breakdown of the QFT description, and therefore sets an upper bound on k_{QFT} . The goal of this section is to determine the scale at which these poles occur as a function of the gravitational parameters and the initial conditions imposed on the top Yukawa coupling.

5.2.1 Analytical analysis

Let us focus on the beta function for the top Yukawa coupling. As we have seen, in the fixed point regime all couplings but y_t tend to zero, so we can neglect their contribution to the beta function β_{y_t} . We can find an approximate analytic solution to the running of y_t in this regime after we ignore the terms that vanish asymptotically. This boils down to solving an equation of the type

$$k \frac{dy_t}{dk} = Ay_t^3 + By_t. \quad (5.7)$$

In our case $A = 9/32\pi^2$ and $B = g_* f_y(\lambda_*)$. First, for simplicity assume that $y_t \gg 1$, and thus we can ignore the linear term since it is sub-leading compared to the cubic one. In this case, we need to solve

$$k \frac{dy_t}{dk} = Ay_t^3. \quad (5.8)$$

Integrating directly the equation and rearranging variables we obtain the solution

$$y_t(k) = \frac{y_t(k_0)}{\sqrt{1 - 2Ay_t^2(k_0) \log\left(\frac{k}{k_0}\right)}}, \quad (5.9)$$

Where we have set the initial condition $y_t(k_0)$ at an arbitrary scale k_0 , not necessarily the Planck scale. As we expected, this solution diverges when the denominator vanishes. This Landau pole occurs at the scale

$$k_{\text{Landau}} = k_0 \exp\left(\frac{1}{2Ay_t^2(k_0)}\right). \quad (5.10)$$

Plugging back the expression of the Landau pole in the solution, we rewrite the solution in terms of it as

$$y_t(k) = \frac{(2A)^{-1/2}}{\sqrt{\log\left(\frac{k_{\text{Landau}}}{k}\right)}}. \quad (5.11)$$

This solution now only depends on the particular value of k_{Λ} . This is precisely the solution one obtains in QED, which is a theory that contains a Landau pole at a very high energy scale. Although this solution is a good approximation, and provides some intuition on the behavior of unsafe trajectories, it is also possible to solve the full Eq. (5.7) and look for the Landau pole. In this case, the condition $y_t \gg 1$ is no longer a requirement for the solution to be valid. Instead, we just need to be on the fixed point regime, to be able to ignore all contribution from the rest of matter couplings. Including the linear term makes obtaining the solution a little more involved, but still has an analytic form, being

$$y_t(k) = \frac{(-B/A)^{1/2}}{\sqrt{1 - \left(1 + \frac{B}{Ay_t^2(k_0)}\right) \left(\frac{k}{k_0}\right)^{-2B}}}, \quad (5.12)$$

where the same initial condition is explicitly present in the solution. Again, we find a Landau pole when the denominator vanishes, which happens at the scale

$$k_{\text{Landau}} = k_0 \left(1 + \frac{B}{Ay_t^2(k_0)}\right)^{\frac{1}{2B}}. \quad (5.13)$$

This Landau pole should correspond to the one we found before when $A \gg B$. In order to prove that this is indeed the case, we can simply take the limit of this expression when $B \rightarrow 0$.

$$\lim_{B \rightarrow 0} k_0 \left(1 + \frac{B}{Ay_t^2(k_0)}\right)^{\frac{1}{2B}} = k_0 \exp\left(\frac{1}{2Ay_t^2(k_0)}\right), \quad (5.14)$$

which is exactly the same expression for the pole that we found before, see Eq. (5.10), as we were expecting. Substituting back the expression for the pole in the solution (5.12), we finally get to

$$y_t(k) = \frac{(-B/A)^{1/2}}{\sqrt{1 - \left(\frac{k_{\text{Landau}}}{k}\right)^{2B}}}. \quad (5.15)$$

This solution now depends on the scale of the Landau pole, and also on the gravitational fixed point values, since $B = g_* f_y(\lambda_*)$. Indeed, k_{Landau} depends on the initial condition $y_t(k_0)$, and on g_* and λ_* , as we can see from (5.13). This scale can take arbitrarily large values, and intuitively, it should diverge when we are approaching the asymptotically

safe trajectory. In other words, the asymptotically safe trajectory has no Landau pole, or it is effectively at infinity. To see this, we recall that the fixed point value corresponds to $y_t^{*2} = -B/A$, meaning that B needs to be negative for y_t^* to be real. We already argued that $B = g_* f_y(\lambda_*) < 0$ for $\lambda_* < -3.28$ as it was required for the existence of the fixed point. Precisely, if we set the initial condition in expression (5.13) to be equal to the fixed point value, $y_t^2(k_0) = y_t^{*2} = -B/A$, then k_Λ diverges for $B < 0$. This means that in fact the Landau pole scale approaches infinity when we are close to the fixed point, as we expected.

5.2.2 Numerical implementation

This analytic results are interesting to understand the behavior of Landau poles, however, they are only valid when $y_t \gg 1$. In reality, we are not going to solve analytically the beta functions, but numerically. The numerical solution, although still approximate because we do not have infinite precision, contains information about the terms we are ignoring in the previous analysis. Therefore, in general, numerical solution will not match exactly the analytic ones. To minimize errors, we will focus on the regime close to the Landau pole, where we know how the coupling should behave.

To actually determine the scale of the Landau pole in our numerical solutions, we will need to find a way to obtain it for a particular trajectory. The idea is the following:

- We rewrite the analytic solution (5.15) as

$$\frac{1}{y_t^2(k)} = -\frac{A}{B} + \frac{A}{B} k_{\text{Landau}}^{2B} \left(\frac{1}{k}\right)^{2B}. \quad (5.16)$$

- Then, we select a set of points (y_t, k) from a particular divergent trajectory. These point must be very close to the divergence in order to capture the running behavior predicted by the analytic solution. We then transform the selected data using the map

$$(y_t, k) \mapsto (1/y_t^2, 1/k^{2B}). \quad (5.17)$$

The value of B depends on the gravitational fixed point values that we used to obtain the solution.

- We perform a linear fit with the transformed data (5.17). We can see from (5.16) that they obey a linear relation in which the intercept is a constant and the slope depends on the Landau pole scale k_{Landau} . Thus, by obtaining the slope we can infer the position of the Landau pole for the numerical solution.

This procedure allows us to extract Landau poles from numerical solutions. We can verify that the results match reasonably well what we expect by looking at the trajectories. One can argue intuitively that increasing the value of the initial condition $y_t(k_{\text{IR}})$ will make the Landau pole appear earlier, i.e. at a lower RG scale. In fact, according to expression (5.13), increasing $y_t^2(k_0)$ results in a decrease of k_{Landau} , as long as we have $B < 0$, which at the same time was the requirement for the existence of the fixed point. As a consequence, the Landau pole scale obtained via the procedure explained above should also decrease if we select a higher initial condition at $k_{\text{IR}} = 173 \text{ GeV}$.

In Fig. 5.5 we plotted the scale of the Landau pole as a function of different values of the running top mas, i.e. different initial conditions in the IR. The values of k_{Landau} are obtained by the numerical procedure detailed previously, it is not an analytical result. The results on Fig. 5.5 confirms the observations we have made so far. First, we see that the Landau pole scale diverges for certain values of the top mass, which are the ones corresponding to an asymptotically safe trajectory. Second, k_{Landau} decreases as we increase the running top mass, as predicted theoretically. Third, for very high values of the initial condition, the Landau pole scale loses the dependence on g_* . We see that the lines start converging to one single line that decays exponentially, according to the behavior predicted by (5.10).

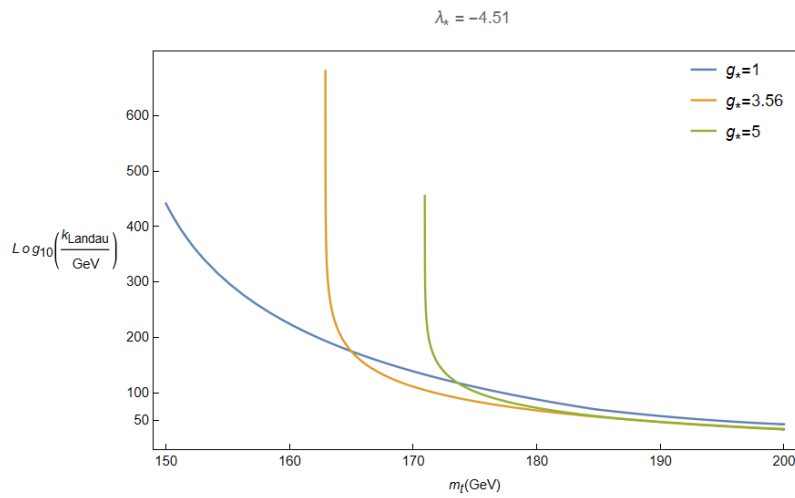


Figure 5.5: Scale of the Landau pole, k_{Landau} , as a function of the running top mass at $k_{\text{IR}} = 173 \text{ GeV}$ for $\lambda_* = -4.51$ and different values of g_* .

We have seen that the analytical method to look for Landau poles and the numerical procedure agree on the conclusions we extracted. Therefore, we found a systematic and consistent way to compute the Landau pole for a particular unsafe trajectory. The Landau pole obtained for a certain initial condition poses a constraint on the maximum value for the scale at which the QFT framework might still be valid, at any generic value of the gravitational parameters. However, this might break down before due to other physical mechanisms, if for example new degrees of freedom of other fundamental theory take control of the flow.

5.3 Interlude: a possible signature of asymptotic safety

A particular feature of the numerical solutions we computed for the running of $y_t(k)$ is the presence of a “bump” around the fixed point scale. This can be seen in figures 4.5 and 5.3, where the solution transitions from a decreasing smooth curve to its constant asymptotic behavior. In between those regimes, it seems like the solution increases for brief range and the decreases again shortly after, resulting in what we call a bump. Although this may be regarded as a small error, product of the numerical integration, it is worth checking if it is an actual physical feature or not. For that, we repeat the integration, keeping the same initial condition $y_t(k_{\text{IR}})$, for different values of g_* . The results are displayed in Fig. 5.6.

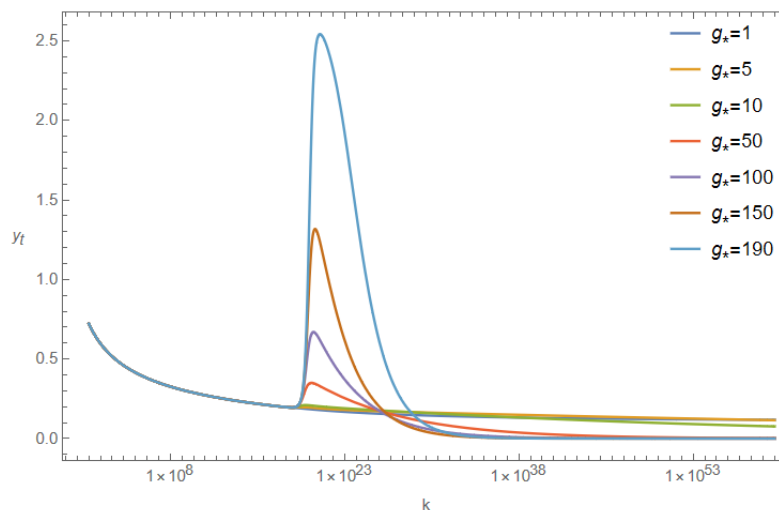


Figure 5.6: Numerical integration of $y_t(k)$ for a range of different values of g_* . It can be seen that the height and position of the “bumps” increases along with g_* .

The numerical solutions in Fig. 5.6 show that the bumps are not due to numerical errors, but are a true feature of the solution. This claim is based on the fact that for large values of g_* the bump is not only still present, but it also gets more pronounced. The explanation for this is that as g_* gets higher, the transition to the fixed point regime occurs more suddenly, and the curve gets steeper. If instead of the approximation (4.10), we had used a Heaviside function, $g(k) = g_*\theta(k - k_{\text{FP}})$, instead of a bump we would have seen a peak, i.e. a continuous solution with discontinuous first derivative. The effect of the sudden growth of g_* on the top Yukawa coupling is what makes the bump appear, however, the growing behavior gets rapidly suppressed as soon as we enter into the fixed point regime.

We can study the position and height of the bump as a function of g_* . For that, we take a sample of values for g_* and examine the bump in each case. The position of the bump is represented in Fig. 5.7, and its height in Fig. 5.8.

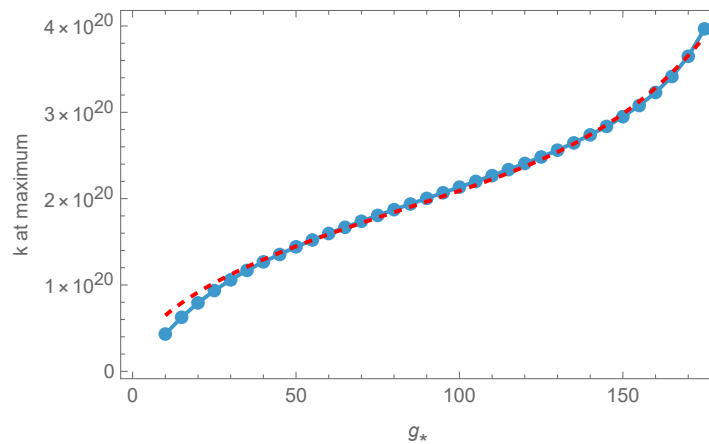


Figure 5.7: RG scale at which the maximum of the bump takes place as a function of g_* . Red dotted line represents a fitting function.

For the position of the maximum, we expect it to be close to the scale of the fixed point, thus $k_{\text{max}} \sim k_{\text{FP}}$. This statement is motivated by the previous discussion on the sudden growth of the gravitational couplings. With the sampled data, we perform a fit to a function of the type $a\sqrt{g_*} + cg_*^d$, with three parameters a , c and d . The result of this fit, represented by a red dotted line in Fig. 5.7 is

$$2.04697 \times 10^{19} \sqrt{g_*} + 9.27624 \times 10^6 g_*^{5.84069}. \quad (5.18)$$

We see a dominant contribution of the term proportional to $\sqrt{g_*}$, with the same order of magnitude of the Planck scale, as expected.

For the maximum in the bump, there is no clear intuition of its behavior with g_* , other than it should be an increasing function. This time, we perform a fit to a function of the type $ag_*^b + cg_*^d$, with four parameters a , b , c and d . The result, plotted as well as a red dotted line in Fig. 5.8, is

$$0.103 g_*^{0.286} + 3.46 \times 10^{-7} g_*^{2.95}. \quad (5.19)$$

The fact that all four parameters are positive confirm that the maximum tends to increase, as we argued. However, this result does not offer any clear interpretation.

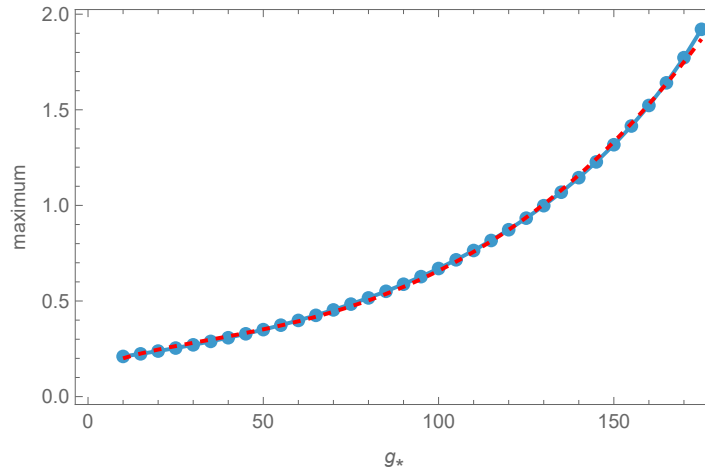


Figure 5.8: Maximum value of $y_t(k)$ in the region of the bump for different values of g_* . Red dotted line correspond to a fitting function.

Although there is not much more to say about the results here, the presence of bumps has phenomenological interest. It has been observed in recent studies that a bump also appears close to the Planck scale in scattering amplitudes in asymptotic safety, see [70, 71, 80]. The authors mention that it could be a graviton bound state resonance, but it is not clear yet. The interesting thing here is that the running of the Newton coupling is manifested in physical observables. Hence, if we were able to run experiments at energy scales close to the fixed point regime, measuring this bumps would be a clear signal that asymptotic safety is realized in nature. Being able to detect this and compare it to the asymptotic safety prediction would be a great success, and it is definitely a thing that it is worth studying in more detail in future works.

5.4 Dependence of the gravitational fixed point on the number of fields

So far we have treated the gravitational fixed point values g_* and λ_* as external parameters in our study to see how these affect the running of the top Yukawa coupling. The previous analysis allowed us to possibly constrain these values. However, as we discussed at the beginning of chapter 4, their value is also affected by the number of matter fields. In this section, we will study how the gravitational fixed point values change with respect to the number of fields that we include. This way, instead of constraining their values, we could constrain the number of fields that our model contains. In particular, since $k_{\text{FP}} = \sqrt{g_*} M_{Pl}$, in a scenario of effective ASQG in which $k_{\text{FP}} < k_{\text{QFT}} < M_{Pl}$ one would need to lower g_* from $g_* \sim \mathcal{O}(1)$ to $g_* \ll 1$. We shall see in particular how this can be done by introducing a large number of matter fields.

To proceed with the analysis, we will take the full beta functions for g and λ , i.e. equations (4.12) and (4.13). The goal is to find the zeroes of these functions depending on the number of matter fields, N_S , N_W and N_V , which will be taken as parameters. Finding analytic solutions is possible, although very involved. The problem they have is that they do not always take real values, due to the presence of square roots appearing in a non-trivial way. As we will see, interesting results can be drawn by solving the algebraic equations numerically, because these will be continuous and well-behaved.

5.4.1 General features

When studying the fixed point equations in the presence of an arbitrary number of minimally coupled scalar, vector, and fermion fields, the fixed point equations become much more complicated. Yet, these equations can be solved analytically, yielding five solutions. One of them is the standard GFP and we will not discuss it further. Then, there are four NGFP solutions, whose dependence on the number of scalars, vectors, and fermions is highly involved. In each of the following subsections, we will refer to them as the first, second, third, and fourth solutions, always keeping the same order. Importantly, the first fixed point solution is, for a small number of fields, the physically relevant one: it is the one that in the limit of zero fields yields the known Reuter fixed point of ASQG and for the case when the number of fields matches those in the SM, one gets the combined asymptotically safe UV-completion. To investigate the complicated

structure of this system, in the following sections we will study the dependence on the number of different types of fields separately.

5.4.2 Dependence on number of scalar fields

First, let us focus only on the dependence on the number of scalar fields. To this end, we set the number of the other types of fields to zero, i.e. $N_W = N_V = 0$. This way, we ignore any possible effect they may have on the fixed point. The analytical solutions to the equations are displayed in Fig. 5.9.

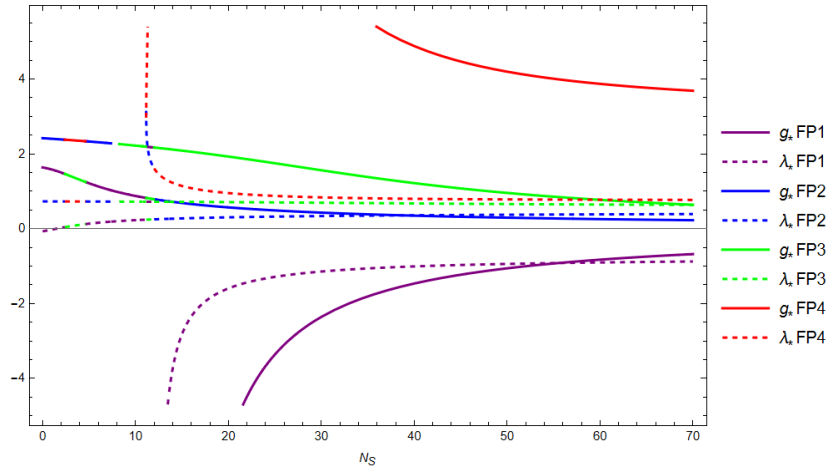


Figure 5.9: Analytic solution for the fixed point values of g_* and λ_* depending on the number of scalar fields N_S . Each color represents a different solution.

What immediately draws attention in Fig. 5.9 is that solutions seem to alternate between each other. One particular solution ceases to exist for a finite range of N_S and then reappears, while the gap in between is filled by another solution, leaving no empty gaps in any solution. One might be tempted to think that this is an error, and that solutions are not displayed correctly. However, there is an explanation for this, and relies on the fact that we are dealing with a polynomial of fourth order. The analytical solution will not be shown here because it is very long and complicated, but can be written without needing any special function. In fact, the roots of a fourth degree polynomial will contain expressions of the type $\sqrt{\dots} \pm \sqrt{\dots}$. Therefore, when one solution becomes complex, other can become real, and vice versa. This way, the analytical results we obtain are generally complex, and we only represent real-valued solution in Fig. 5.9. In any way, since the solutions look continuous when joined together, we can then consider an “effective” solution, formed as a combination of different analytical solutions.

We can take another approach. Instead of finding the analytical expressions for the roots of the beta functions, let us look for numerical solutions, not caring about any identification with the previous analytical solutions. In this case, we look for solutions at every value of N_S and order them from lowest to highest value. By taking a numeric approach, one loses the ambiguity given by the “jumps” in the analytical solutions, leading to smooth solutions, not only in the fixed point values but also in the critical exponents. If we look at Fig. 5.9, we see that out of the four solutions that we can find in general, we can only identify three “effective” solutions that satisfy $g_* > 0 \forall N_S$, while the other one becomes negative. We disregard solutions containing $g_* < 0$, since, as we explained in previous chapters, these are not physical. Thus, we are left with three possible solutions. We display the three effective solutions in Fig. 5.10, together with their associated critical exponents that we computed numerically as well.

As we can see in Fig. 5.10, only the first effective fixed point seems to be physically relevant. The other two solutions feature one large critical exponent, $-\theta \gtrsim \mathcal{O}(100)$, defining an irrelevant direction, while the first one is fully UV-attractive. This can be seen in Fig. 5.10b, where we observe that the critical exponents have both positive real parts, and become complex conjugated for a finite range of N_S . Furthermore, the third fixed point does not even seem to be defined for a small number of scalar fields, $N_S \lesssim 20$, either diverging or being complex-valued.

The analytical expressions are still useful, because we can use them to make an expansion for $N_S \rightarrow \infty$ and analyze the asymptotic behavior of the solutions. If we do this, the results are:

- For the first fixed point

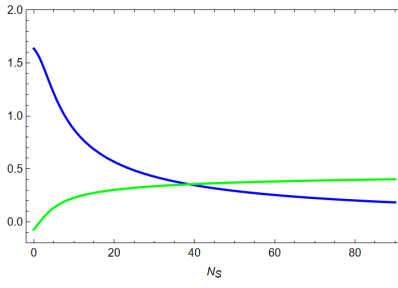
$$g_* = \frac{8\pi}{N_S} + \mathcal{O}\left(\frac{1}{N_S}\right)^{3/2}, \quad (5.20)$$

$$\lambda_* = \frac{1}{2} - \frac{1}{\sqrt{N_S}} + \mathcal{O}\left(\frac{1}{N_S}\right). \quad (5.21)$$

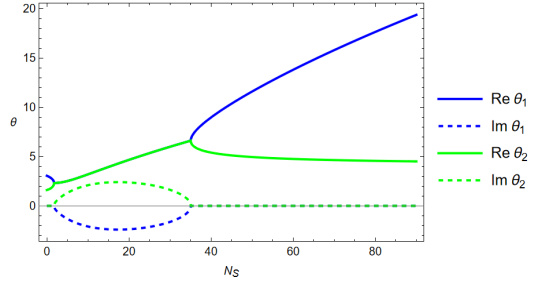
- For the second fixed point

$$g_* = \frac{8\pi}{N_S} + \mathcal{O}\left(\frac{1}{N_S}\right)^{3/2}, \quad (5.22)$$

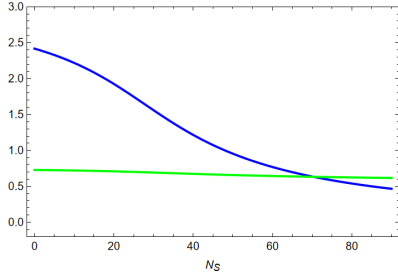
$$\lambda_* = \frac{1}{2} + \frac{1}{\sqrt{N_S}} + \mathcal{O}\left(\frac{1}{N_S}\right). \quad (5.23)$$



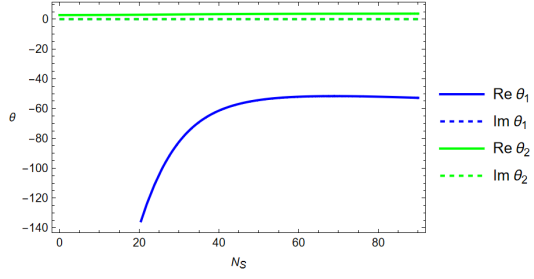
(a) Values of g_* and λ_* for the first fixed point.



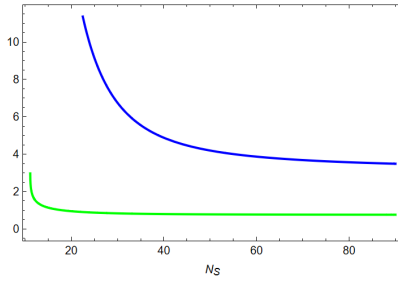
(b) Critical exponents for the first fixed point.



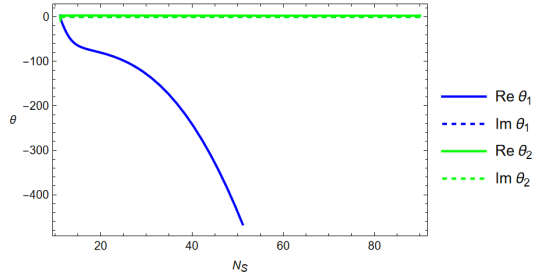
(c) Values of g_* and λ_* for the second fixed point.



(d) Critical exponents for the second fixed point.



(e) Values of g_* and λ_* for the third fixed point.



(f) Critical exponents for the third fixed point.

Figure 5.10: Dependence on the number of scalar fields N_S , when setting $N_W = N_V = 0$, of the effective gravitational fixed points and their corresponding critical exponents.

- And for the third fixed point

$$g_* = 3.08064 + \frac{24.1968}{N_S} + \mathcal{O}\left(\frac{1}{N_S}\right)^2, \quad (5.24)$$

$$\lambda_* = 0.75 + \frac{0.75}{N_S} + \mathcal{O}\left(\frac{1}{N_S}\right)^{3/2}. \quad (5.25)$$

With this, if we consider the first fixed point as the physically relevant one, we conclude that $g_* \sim 1/N_S$. This implies that the scale of the fixed point decreases with the number of scalar fields as $N_S^{-1/2}$, $k_{\text{FP}} = \sqrt{g_*} M_{\text{Pl}} \sim M_{\text{Pl}}/\sqrt{N_S}$. Interestingly, this coincides with the notorious “species scale” of string theory [37], namely $M_{\text{sp}} = M_{\text{Pl}}/\sqrt{N_{\text{sp}}}$ with

N_{sp} the number of species. Another thing to point out is that, while g_* decreases with N_S and approaches zero, we cannot approach a perturbative regime due to the cosmological constant approaching a constant value of 0.5, making the system non-perturbative regardless of the number of scalars we add. Similar features will be found in the following.

5.4.3 Dependence on number of vector fields

We repeat now the analysis of the previous section, but this time for the number of vector fields. For that, we set again the rest of fields to zero, $N_S = N_W = 0$, and find the analytical solutions. The result of this is displayed in Fig. 5.11

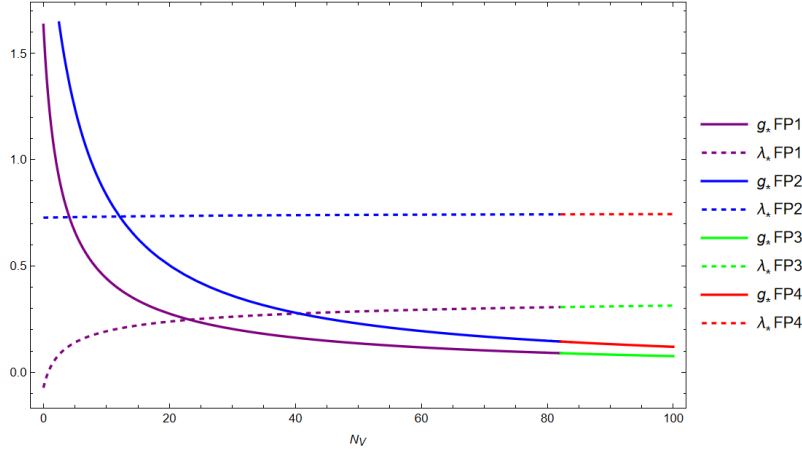


Figure 5.11: Analytic solution for the fixed point values of g_* and λ_* depending on the number of vector fields N_V . Each color represents a different solution.

In this case, we also see that solutions are not always real, but they add up to effective solutions. In the plotted range in Fig. 5.12, two solutions are real until $N_V \approx 80$, where they become complex and the other two solutions replace them being real. We cannot make any further claim, since it is not easy to predict if the first two solutions will eventually go back to being real or not.

This time, there seem to be just two effective fixed points, so we proceed to compute them numerically along with their corresponding critical exponents. The results are displayed in Fig. 5.12

Not surprisingly, again the first fixed point seems to be the one that is physically relevant. The reason is the same as in the previous section: it is the only fixed point

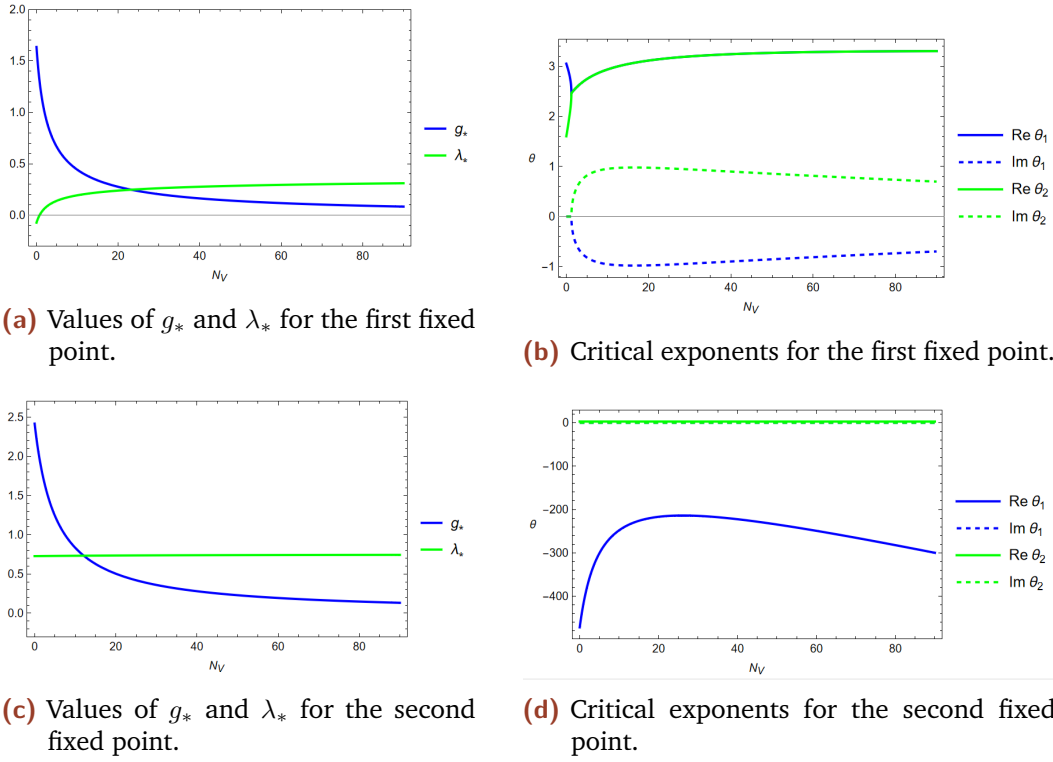


Figure 5.12: Dependence on the number of vector fields N_V , when setting $N_S = N_W = 0$, of the effective gravitational fixed points and their corresponding critical exponents.

that is fully UV-attractive and with reasonable values for the critical exponents. Again, we see that the critical exponents are complex conjugated with positive real parts, see Fig. 5.12b. Since both fixed point tend to approach the GFP for large values of N_V , we then expand their analytical expressions around $N_V \rightarrow \infty$, finding that:

- The first fixed point behaves as

$$g_* = \frac{3\pi}{N_V} + \mathcal{O}\left(\frac{1}{N_V}\right)^2, \quad (5.26)$$

$$\lambda_* = 0.375 - \frac{15.0949}{N_V} + \mathcal{O}\left(\frac{1}{N_V}\right)^{3/2}. \quad (5.27)$$

- The second fixed point behaves as

$$g_* = \frac{4\pi}{N_V} + \mathcal{O}\left(\frac{1}{N_V}\right)^2, \quad (5.28)$$

$$\lambda_* = 0.75 - \frac{0.75}{N_V} + \mathcal{O}\left(\frac{1}{N_V}\right)^{3/2}. \quad (5.29)$$

The same asymptotic behavior as the scalar fields applies to vectors. We have that for the physically relevant fixed point, the dimensionless Newton coupling scales as N_V^{-1} , and thus the fixed point scale will decrease if we add more vector fields, $k_{\text{FP}} \sim M_{\text{Pl}}/\sqrt{N_V}$. Although the vector fields make g_* to become smaller, λ_* approaches asymptotically a constant value of 0.375, not allowing the system to become more perturbative, even if we add a large number of fields.

5.4.4 Dependence on number of Weyl fermions

The only type of field remaining in our study is the behavior of the fixed point with respect to the number of Weyl fermions. Once more, we set other numbers of fields to zero, $N_S = N_V = 0$, and we compute the analytical solutions, shown in Fig. 5.13.

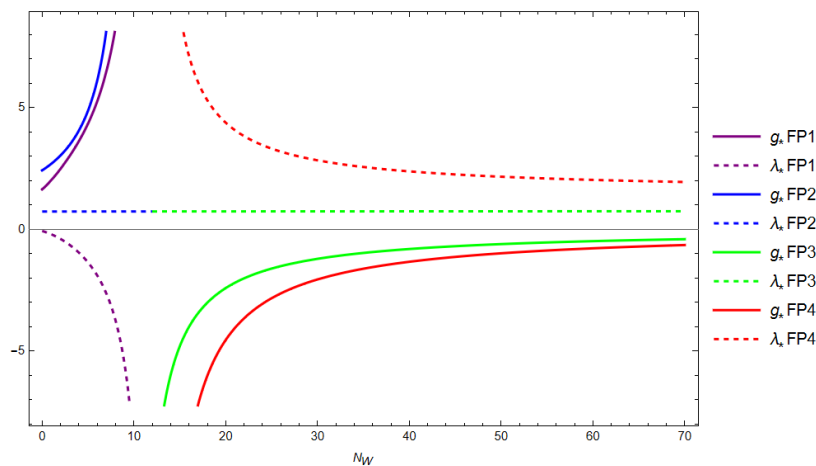


Figure 5.13: Analytic solution for the fixed point values of g_* and λ_* depending on the number of Weyl fermions N_W . Each color represents a different solution.

This time, we find a divergence at a finite number, $N_{W,\text{div}} \approx \mathcal{O}(10)$. Only two real solutions are found before the divergence, and after that, they become complex. We still find another two real solutions after $N_{W,\text{div}}$, however, these have $g_* < 0$ and thus are not physical. Therefore, in this case it does not make any sense to evaluate the limit $N_W \rightarrow \infty$. A way to push further the number of Weyl fermions at which the divergence takes place, is by adding other bosonic fields. This is precisely why we have a fixed point with $g_* > 0$ for the SM, that has a large number of fermions, $N_W = 45$. Regarding the two effective fixed points that we seem to have, they are displayed in Fig. 5.14.

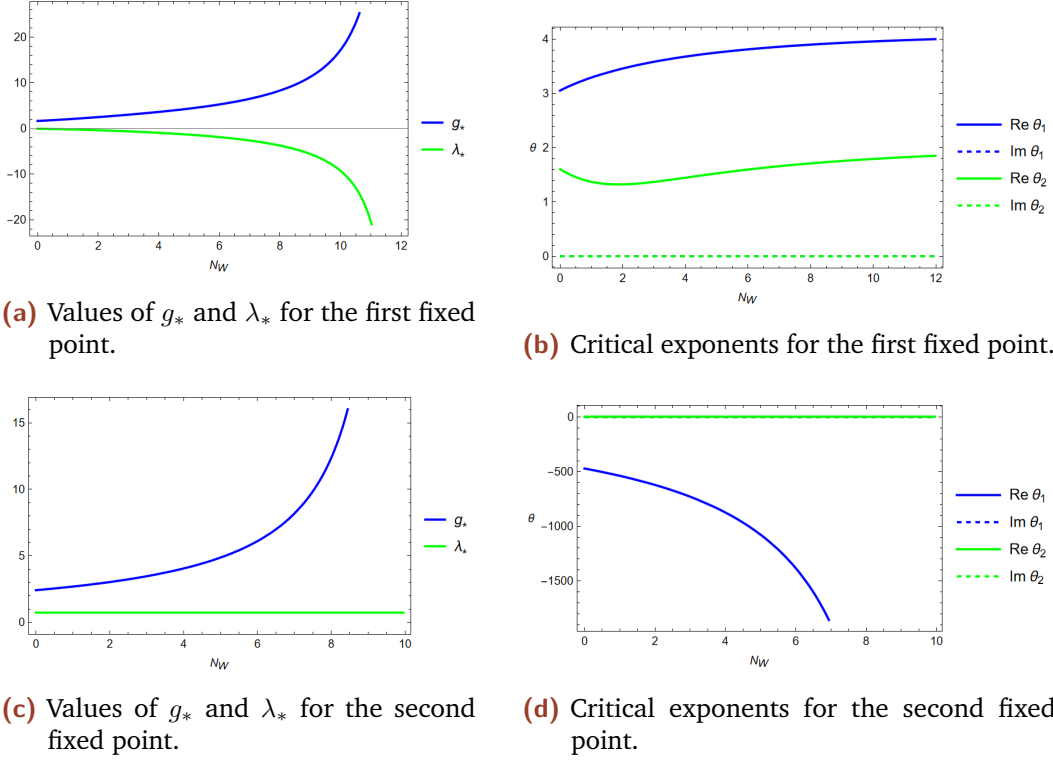


Figure 5.14: Dependence on the number of Weyl fermions N_W , when setting $N_S = N_V = 0$, of the effective gravitational fixed points and their corresponding critical exponents.

In the same way as the other cases we studied before, we have that the fixed with the lower values for g_* corresponds to the physically relevant effective fixed point. This time, both eigenvalues are real for all the domain in which the fixed point is defined. They are also positive, as corresponds to a fully UV-attractive fixed point.

5.4.5 Interlude: scales in string theory

Let us stop for a moment and comment on some aspects of string theory that are relevant for our purposes and allow us to compare different scenarios with ASQG. For the basic concepts, we will follow the logic presented in the chapter on string theory in the lecture notes [37].

In string theory, there is a characteristic scale, namely the string scale, M_S , that is associated with the extended nature of strings, as we commented at the beginning of chapter 4. This scale is one example for what we call k_{QFT} . When strings are weakly coupled, this scale is much smaller than Planck scale, $M_S \ll M_{\text{Pl}}$, and thus new physics shows up before expected, as we were discussing. We can still get an upper bound on this scale, which is the so-called *species scale*, that we mentioned in section 5.4.2.

The species scale is in particular the highest possible scale in which one can use the EFT, before entering the UV-completion regime. In the context of string theory, when strings are weakly coupled, it coincides with the string scale. Then, in summary we have that [140],

$$M_S \lesssim M_{\text{sp}} \lesssim \frac{M_{\text{Pl}}}{\sqrt{N_{\text{sp}}}} \lesssim M_{\text{Pl}} \quad (5.30)$$

Since in the context of ASQG the scale at which one enters the UV-completion is the fixed point scale, it is interesting that, as we have seen in section 5.4.2 and 5.4.3, it precisely follows the same asymptotic behavior as the species scale in string theory, $k_{\text{FP}} \sim M_{\text{Pl}}/\sqrt{N_{\text{sp}}}$. Hence by enlarging the number of fields in ASQG, one would recover a (partially) weakly coupled regime and some of the features of string theory. We note, however, that this would require violating the bound in section 5.1, which guarantees a top mass compatible with experiments.

5.4.6 Existence of a physically relevant fixed point

After having studied the effect of different matter fields on the fixed point, we can draw some conclusions. First, for every type of matter field, there is one physically relevant gravitational NGFP which is fully-attractive. Second, while scalars and vectors make g_* decrease and approach zero asymptotically, fermions have the opposite effect, they increase the value of g_* and eventually make it diverge, posing a maximum number of fermion fields that we can add before we destroy the fixed point. Third, only g_* can vanish asymptotically, while λ_* tends to a constant value, making it impossible for the interacting fixed point to lie arbitrarily close to the GFP. In summary, it is possible to find a reliable Reuter fixed point in gravity-matter systems for a large number of different configurations of matter fields. However, not every combination of fields is allowed, delimiting a region in the space (N_S, N_V, N_W) for the existence of this point.

Finally, to answer the question of how the domain of existence for the fixed point looks like, we plot in Fig. 5.15 the allowed region we were discussing. We project this region in a two-dimensional plane, where the axes correspond to the combination of number of fields $(N_W - N_V - N_S/2)$ and $(N_W - 4N_V + N_S)$ respectively. We choose this particular coordinates because the beta functions (4.12) and (4.13) contain explicitly these precise combinations of fields. The region plotted in purple in Fig. 5.15 is where it is possible to find the physically relevant fixed point. Other fixed points may admit a larger or smaller region, but we focus on the solution which is fully UV-attractive

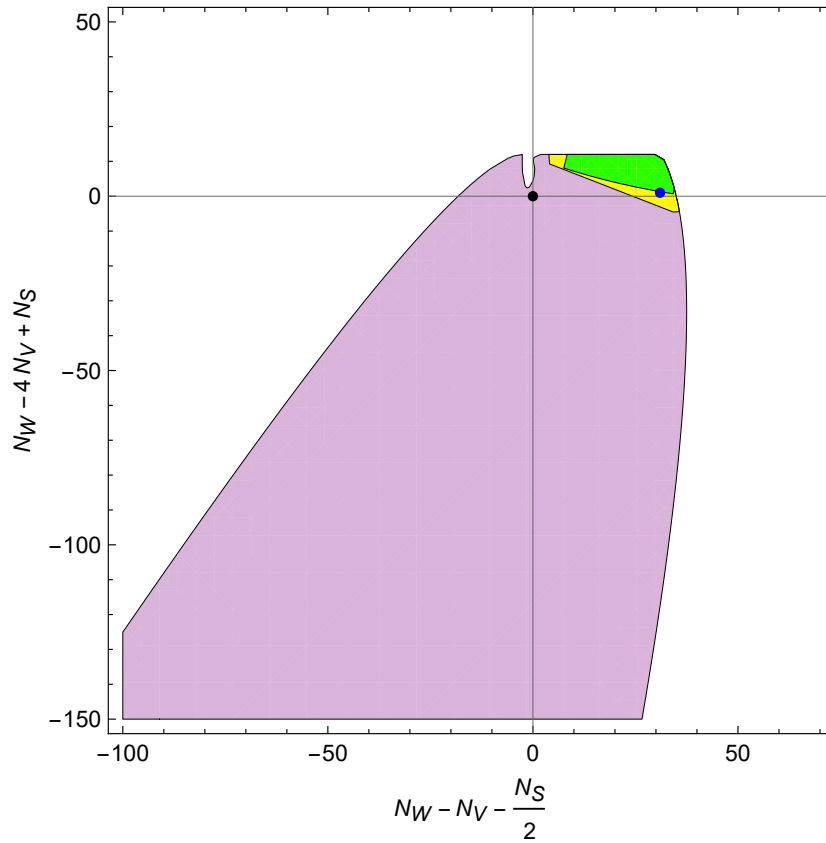


Figure 5.15: Purple region: existence of a physically relevant fixed point. Yellow region: subset of the purple region where we have $\lambda_* < -3.28$, and thus it is possible to retrodict the top mass. Green region: subset of the yellow region where the post-diction of the top mass obtained from such fixed point can be compatible with experimental data. The black dot corresponds to pure gravity, $N_S = N_V = N_W = 0$. The blue dot corresponds to the SM, $N_S = 4$, $N_V = 12$, $N_W = 45$.

and whose critical exponents are not too large. As we can see, the region extends to large numbers of values, but only where the coordinates are negative, spanning an infinite area. The reason for that is because both coordinates contain the number of vector fields with a negative sign. We know that we can add arbitrarily large number of vectors without destroying the fixed point, and the same happens with scalars. On the other hand, vectors appear in the coordinates with a positive sign, and we saw how these tend to destroy the fixed point if we add too many. This explains the shape of the purple region, and why it is only bounded in the positive direction in the chosen coordinates.

The origin of our coordinates in Fig. 5.15 corresponds to the case of pure gravity, i.e. no matter fields included, and lies in the allowed region. The yellow region that appears in the plot corresponds to a subset of coordinates that not only yield a valid fixed point, but also result in a fixed point value for the cosmological constant that is

$\lambda_* < -3.28$, the necessary condition to be able to retrodict the top mass. Fortunately, if we plug in the matter content of the SM, that point corresponds to the yellow region. This is not a surprise, because we have already seen an example in chapter 4 of how we can obtain the top mass when considering only SM fields, but it is nonetheless somewhat miraculous that the SM sits precisely on this small region compared to all the possibilities. Then, a valid post-diction of the top mass would be achievable for small variations of the number of fields. This means that there exist the possibility of adding more fields to the SM and modify slightly the retrodicted value for the top mass, and perhaps make it compatible with experiments, as we have already conjectured. Indeed, a smaller subset of the yellow region, i.e. the green region in Fig. 5.15, corresponds to values of the gravitational parameters that lead to the possibility of predicting a top mass that is compatible with experimental data, either in safe or free trajectories, as we commented on section 5.1. This is a remarkable result, because the fact that this region exists confirms that it is possible to find a configuration of fields in which the gravitational fixed point realizes a UV-completion that is compatible with experimental data. Unfortunately, the matter content of the SM does not sit on this green region, as we know from the explicit computation of the top mass in chapter 4, but it seems to be really close to the boundary. This result poses a strong restriction on the number of fields if we are in a scenario where it is possible to retrodict the top mass, and the restrictions get even stronger if we ask for compatibility with experiments.

Conclusions

We began this work by pointing out the problem that gravity has when we try to quantize it, namely that GR is a perturbatively non-renormalizable QFT. Different approaches have tried to find a consistent theory of quantum gravity, each one with some strengths and weaknesses. A solid candidate to do this is ASQG, that we have discussed in detail. Although there are still open questions (as explained in section 2.4), we pointed out the vast evidence for the existence of an interacting fixed point, not only in simple setups, but also in different higher-derivative truncations and gravity-matter systems, see chapters 2 and 3. In particular, including gravity in the SM provides an answer to riddles in particle physics and tames the divergences that we find in such a theory, leading to a UV-completion of gravity and the SM [34]. It has been shown in the literature that when coupling quantum gravity with matter, one gains an enhanced predictive power. In particular it is possible to post-dict the top mass [9] thanks to the existence a particular fixed point that allows us to obtain the running top mass at an experimental scale, $k_{\text{IR}} = 173 \text{ GeV}$, as a byproduct of constructing an asymptotically safe trajectory for the top Yukawa coupling.

6.1 A tale of scales in quantum gravity

The prediction, or rather post-diction, of the top mass has been the central argument for discussing scales. Its particular value is sensitive to the other free parameters of the SM, but more interestingly, in asymptotic safety it is related to the scale of quantum gravity. This scale involves the fixed point value of the dimensionless Newton coupling, $k_{\text{FP}} = \sqrt{g_*} M_{\text{Pl}}$, and corresponds to the transition to the fixed point regime. In the standard calculation in the literature leading to the post-diction of the top map, this scale is set to be the Planck scale. In a scenario in which asymptotic safety is not fundamental though, the fixed point scale may be different than the Planck scale. Reversing the logic, in this this work we studied how the experimental constraints on the top mass bound the fixed point scale, and more generally, how the RG flow constraints the different quantum gravity scales. This naturally led to the generalization of the result in [9] for any arbitrary choice of the gravitational fixed point values, g_*

and λ_* . The result we found is that requiring compatibility with experimental data poses constraints on these parameters. In particular, there is a lower bound for the scale of the fixed point, for which we cannot find trajectories that are at the same time UV-complete and compatible with data. This lower bound is $k_{\text{FP}} > 1.62 M_{\text{Pl}}$. This may be tested against predictions from other approaches, e.g. string theory. If we assumed string theory to be weakly coupled, then the characteristic scale of string would be much below the Planck scale. This means that string theory would excite new degrees of freedom that control the flow of the couplings way before we enter in a fixed point regime, implying that either ASQG is realized at a fundamental level or the fundamental theory has to pick the right initial conditions for the RG flow, without relying on the asymptotically safe trajectory (which instead in an effective scenario would act as an attractor for RG trajectories stemming from the fundamental theory).

In this thesis we also asked ourselves about constraints on the scales at which the QFT description is replaced by another more fundamental theory. This will happen at a scale k_{QFT} , and the only way that we can constrain it is by finding the scale of Landau poles, k_{Landau} . This Landau poles pose upper bounds on the scale at which new physics take over, $k_{\text{QFT}} \leq k_{\text{Landau}}$, since by definition it is the scale at which QFT stops making sense. We studied this by analytical and numerical methods, and found out that this scale depends on the gravitational parameters g_* and λ_* , as well as the initial condition for the top mass in the IR, $m_t(k_{\text{IR}} = 173 \text{ GeV})$. Furthermore, as we should expect, this scale diverges for asymptotically safe trajectories, corresponding to scenarios where the theory is already UV-complete. In those cases, we cannot say anything about new physics, but this might still break down at some earlier scale.

A related question we investigate is how one could possibly change the fixed point scale. Namely, assuming that asymptotic safety is effective, and since the fixed point scale is set by g_* , what can we change in our system to lower or raise the fixed point scale? In this thesis we described this mechanism in detail: the answer to this question is related to the coupling of gravity with matter. Concretely, we have shown that the scale of the fixed point is tightly connected to the number of fields in our theory. In our definition of k_{FP} , this depends on g_* , and this in turn depends on the matter content, specifically the number of fields. Thus, it is interesting to study how the different types of matter fields affect the scale of the fixed point, and relate it to the previous results as well as to known scales in the literature. As argued in chapter 3 and previously studied in the literature [59], it is possible to add a large number of fields and still find a suitable NGFP. However, not any combination of number of fields is valid. A reason

for this is that the screening effect of fermions tends to destroy the fixed point, so in order to preserve it we need to balance it with the contribution of bosonic fields. At the end of chapter 5 we found that the existence of a viable fixed point is constrained within a region of the space of number of fields. This region is actually unbounded, because there is no expected limit for the number of vector fields that one can add. However, inside this large region we find a bounded smaller region in which the fixed point satisfies $\lambda_* < -3.28$, i.e. the necessary condition to be able to retrodict a top mass. Interestingly, we found that the matter content of the SM lies precisely in this subset. Finally, inside this bounded region, we found an even smaller region for which the post-diction of the top mass that can be drawn from those combinations of fields yields a compatible result with experiments. This connects to our previous discussion, because we managed to translate the bound on the fixed point scale to a bound on the possible number of fields that we can have to satisfy compatibility with data while keeping the theory UV-complete. A key result of the analysis is that since asymptotically $g_* \sim 1/N$, in the presence of matter the fixed point scale coincides with the species scale of string theory. Moreover, this also gives us an answer to our question: to lower the fixed point scale of asymptotic safety from the Planck scale down to the string scale, one would need to introduce a large number of matter fields.

6.2 Outlook

Conveniently, the matter content of the SM lies in the subset where it is possible to retrodict a top mass (i.e. where $\lambda_* < -3.28$), and it yields a result compatible with experiments within the systematics of our computations, but it is really at the boundary of the region of compatibility. Since small deviations from the SM can also be inside this region, and the post-diction of the top mass for the SM was already very close to the experimental value, it would be possible to modify the matter content and find another top mass. We conclude then that adding a small number of fields to the model could modify the fixed point and consequently might render the retrodicted top mass closer to the central value provided by experiments. We have not found yet a configuration that fulfills this, but it is natural to believe in this possibility. Natural extensions of the SM that can potentially do this would be the addition of three right-handed neutrinos (3 more Weyl fermions), that would explain neutrino oscillations. Besides, fermions increase the value of g_* , which is what we need to make the result of the top mass closer to experimental data. Other possible extensions include the addition of scalar fields, identified with axion-like particles that can at the same time explain the presence of dark matter.

Despite the success of ASQG in providing an explanation for the top mass, among other achievements, we still do not know if it constitutes a fundamental theory or not. In a scenario where the QFT framework breaks down, i.e. the RG flow needs to be described by new physics, at a much larger scale than the fixed point scale, $k_{\text{QFT}} \gg k_{\text{FP}}$, it still makes sense to find trajectories that UV-complete the theory. However, if QFT breaks down at sub-planckian scales, then there is no reason to choose one particular initial condition over a different one, and trajectories that are predicted to be divergent at scales $k > k_{\text{FP}}$ may be affected by unknown physics that regularize the flow. In this latter scenario, we would say that asymptotic safety is *effective*. The scale k_{QFT} , regardless of the presence of Landau poles, must come from other possible more fundamental theories, such as string theory or something else. For other different theories, we can compare it with the ASQG constraints proposed in this work and draw the consequent conclusions.

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