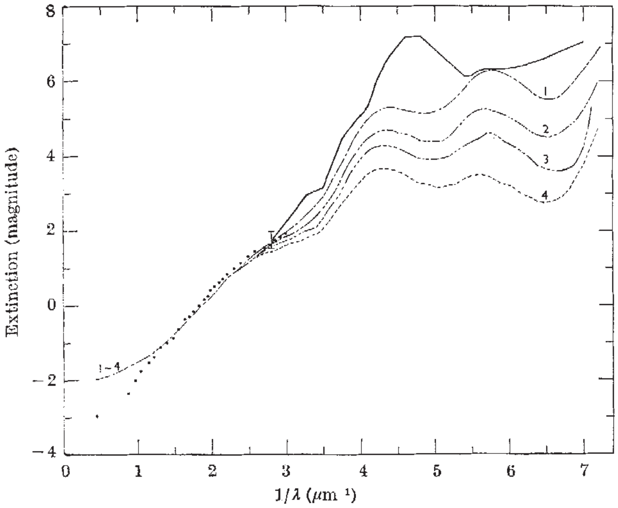

Fig. 2. Reddening curves of ground based observations (filled circles) and the measures of Stecher[2] (solid curve), compared with predictions for extinction by compound graphite/silicate particles. Curves 1 to 4 correspond to differing mixtures giving 4×, 5×, 6× and 8× the extinction at 4500 Å by graphite as compared with silicate.

Additional support for this view can be found in the data on the measurements of the diffuse galactic radiation now available over a wide wavelength range. Van de Hulst and de Jong[12] have compared theory with Witt's[13] observations of diffuse galactic light in the Cygnus region in U and B colours. In both colours a strong enhancement of surface brightness in the galactic plane indicates a high value of albedo, α, of interstellar dust particles. The lower limit has been set at $\alpha \geq 0\cdot5$. For this value of the albedo there should be a measurable component of diffuse galactic light in the ultraviolet ($\lambda < 3000$ Å). Lillie and Witt[14] have reported that there is no evidence of diffuse galactic light in the wavelength range 2100 Å–2800 Å, indicating a low albedo of interstellar dust particles at these wavelengths, a figure of $0\cdot2 \pm 0\cdot2$ being derived. But further into the ultraviolet, for example, $\lambda < 2100$ Å, a high value of α has been obtained[15].

A strong interaction of the starlight with interstellar dust grains in the rocket ultraviolet becomes evident from the recent measurements of interstellar extinction, but the very low value of the albedo in the narrow spectral range 2100–2800 Å as compared with its values at other wavelengths implies that the energy of the star light in this wavelength region is truly absorbed. This adds additional weight to our suggestion that a broad interstellar absorption feature occurs close to 2200 Å.

K. NANDY
H. SEDDON

Royal Observatory,
Edinburgh.

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Kerr Metric Black Holes

LYNDEN-BELL has suggested[1] that a massive collapsed object may lie at the centre of many galaxies, including our own, and that a continuing flux of matter into such a collapsed object, or "black hole", may be the source of energy for the non-thermal radiation observed to come from some galactic nuclei. The metric used to describe the geometry of space-time in the vicinity of the collapsed object in this and other papers[2] on collapsed objects in astrophysics has been the spherically symmetric Schwarzschild metric, which is valid only if the collapsed object has zero angular momentum. In fact, one would expect any such collapsed object to possess considerable angular momentum, because a great deal of angular momentum must be lost from a typical galactic nucleus to permit collapse inside the gravitational radius in the first place.

Very probably the metric describing a collapsed object with angular momentum is the Kerr metric[3], a stationary, axially symmetric solution to the vacuum Einstein equations with two free parameters, m and a. Assigning these parameters the dimensions of length, they are related[4] to the total gravitational mass M and the total angular momentum J of the black hole by $m = GM/c^2$, $a = J/cM$. Written in a form[5] which reduces to the standard form of Schwarzschild metric when $a = 0$, the Kerr metric is

$$ds^2 = \frac{\rho^2}{\Delta}dr^2 + \rho^2\,d\theta^2 + \frac{B}{\rho^2}\sin^2\theta\left(d\varphi - \frac{2amrc}{B}\,dt\right)^2 - \frac{\Delta\rho^2}{B}c^2\,dt^2 \quad (1)$$

where Δ, ρ^2 and B are defined by

$$\Delta = r^2 - 2mr + a^2$$
$$\rho^2 = r^2 + a^2\cos^2\theta$$
$$B = (r^2 + a^2)^2 - \Delta a^2 \sin^2\theta$$

If $a^2/m^2 = (cJ/GM^2)^2 < 1$, there is an event horizon at $r = r_+ = m + (m^2 - a^2)^{1/2}$ of qualitatively the same sort as the one at the gravitational radius $r = 2m$ in the Schwarzschild metric. At the event horizon $\Delta = 0$, but ρ^2 and B are finite. As for the Schwarzschild metric, the singularity in the line element (1) is just a coordinate singularity[5] due to the fact that the t = constant hypersurfaces change from being space-like when $\Delta > 0$ to time-like when $\Delta < 0$. An analysis of perturbations of the Schwarzschild metric[6] has shown that to second order in cJ/GM^2 the Kerr metric is the only stationary, asymptotically flat vacuum metric which has a non-singular event horizon.

This fact strongly suggests that the irreversible gravitational collapse of a rotating body inside a trapped surface[6] will result in the exterior metric in the region outside the trapped surfaces asymptotically approaching the Kerr metric as the transient gravitational waves die down. Additional evidence comes from calculations by myself and Wagoner[7], and shows that in quasi-stationary collapse of a uniformly rotating disk the exterior metric in the region $r > m$ becomes the Kerr metric with $a^2/m^2 = 1$ as the redshift from the centre of the disk becomes infinitely large.

I now argue that the most appropriate value to take for a^2/m^2 in representing a collapsed object in a galactic nucleus is a value only slightly less than one. If the black hole results from a collapse of a rotating body during which the centrifugal force becomes comparable with the gravitational force, as seems likely, one would not expect cJ/GM^2 to be small compared to one initially. After the initial collapse there is likely to be continued accretion of matter, at least if the black hole is in a galactic nucleus. I assume that this accretion will take place from a disk rotating in the equatorial plane of the collapsed object, as proposed by Lynden-Bell[1]. An accreting matter element slowly loses angular momentum in the disk due to torques exerted by magnetic fields until it reaches the radius of the innermost stable orbit in the equatorial plane. At this point it falls into the black hole without further loss of energy and angular momentum. The energy per unit rest mass released in this process is one minus the energy per unit mass of a test particle in the innermost stable circular orbit. The total gravitational mass and angular momentum of the black hole are changed by the energy per unit mass and angular momentum per unit mass of the innermost stable circular orbit, times the rest mass accreted. The result of this change in the parameters of the black hole is to make $a^2 = m^2$ after a finite amount of matter has been accreted, even starting from $a = 0$, the Schwarzschild metric.

Solutions to the geodesic equations for the Kerr metric have been obtained by de Felice[8] and Carter[9]. If $z = r/m$, E is the energy per unit mass, and Φ is the angular momentum per unit mass for the innermost stable circular orbit, these quantities are related by

$$E \measuredangle \left[1 - \frac{2}{3z}\right]^{\frac{1}{2}} c^2, \quad \Phi = \frac{2mc}{3\sqrt{3}} (1 + 2\,(3z - 2)^{\frac{1}{2}}) \qquad (2)$$

and the value of a/m for the black hole is

$$a/m = \frac{1}{3} z^{\frac{1}{2}}\, [4 - (3z - 2)^{\frac{1}{2}}] \qquad (3)$$

The parameter z ranges from 6 for $a/m = 0$ to 1 for $a/m = 1$.

There is a simple analytic solution for the variation of z with m as mass is accreted,

$$z/z_1 = (m_1/m)^2$$

In terms of the rest mass accreted, Δm_0, from the time when $m = m_1$,

$$\frac{m}{m_1} = \left(\frac{3z_1}{2} - 1\right)^{\frac{1}{2}} \sin\left[\left(\frac{2}{3z_1}\right)^{\frac{1}{2}} \frac{\Delta m_0}{m_1}\right] + \cos\left[\left(\frac{}{}\right)^{\frac{1}{2}} \frac{\Delta m_0}{m_1}\right] \qquad (4)$$

Thus if initially $a = 0$ and $m = m_1$, the parameters of the black hole become $a = m = 6^{1/2} m_1$ after a rest mass $\Delta m_0 = 3m_1\ [\sin^{-1}\ (2/3)^{1/2} - \sin^{-1}\ 1/3]$ has been accreted. Further accretion will keep a equal to m. Because not all the matter and radiation will be accreted from orbits with the largest amount of angular momentum consistent with trapping, the actual limiting value of a/m will be less than one by an amount depending on the detailed structure and radiation pattern of the disk surrounding the black hole.

If the accreting matter does not all have positive angular momentum (taking $a > 0$) the argument fails, because the cross-section for trapping accreting matter is larger for matter with negative angular momentum than for matter with positive angular momentum. In a galactic nucleus the matter can be expected to have angular momentum from the rotation of the galaxy.

The likelihood that a/m will be close to one alters the model of accretion proposed by Lynden-Bell both quantitatively and qualitatively. While in the Schwarzschild case the departures from flat space are moderate in the vicinity of the disk at $r \gtrsim 6m$, the geometry and propagation of light are very different from flat space at $r \approx m$ in the case $a = m$. One important difference is that the energy release per unit rest mass of accreted matter can be as large as $0{\cdot}432c^2$, almost an order of magnitude larger than the value of $0{\cdot}057c^2$ when $a = 0$.

When $a = m$ it is convenient to use as a radius variable $\xi = r/m - 1$. The finite coordinate distance $\xi = 0$ to $\xi = 1$ corresponds to an infinite proper radial distance, the proper radius being proportional to $\ln\,\xi$ as $\xi \rightarrow 0$. Thus a large fraction of the mass of the disk surrounding the black hole may be located at $\xi \ll 1$.

As measured by an observer comoving with the matter in a direct ($\Phi > 0$) circular orbit at $\xi = \xi_0 \ll 1$, photons emitted into the forward hemisphere relative to the velocity of rotation escape, but most of the photons emitted into the backward hemisphere are trapped by the black hole. Almost all of the light emitted by the matter at $\xi_0 \ll 1$ which does escape comes out at an asymptotic polar angle θ in the range $|\pi/2 - \theta| \leq \sin^{-1}\ [2\sqrt{3} - 3]^{1/2}$ and with an impact parameter $2m/\sin\theta$ with respect to the axis of symmetry. The distant observer sees a spot of light, perhaps drawn out into a line parallel to the axis of symmetry, containing frequency shifts from large redshifts to moderate redshifts or even blueshifts if the observer is near the equatorial plane. The ratio of observed to emitted frequency ranges from the order of ξ_0 to at most $\sqrt{3}$. The escaping photons gradually spiral outward, intersecting the disk many times, if they start from $\xi_0 \ll 1$. This obviously makes the transfer of energy and radiation in the vicinity of the disk very complex.

The internal dynamics of the disk, the rate of generation of magnetic field due to twisting of field lines by differential rotation and the decay by field annihilation and generation of high energy particles, depend on the local rate of shear of the matter in the disk. The angular velocity of the matter is $\Omega \simeq \frac{c}{2m}\left(1 - \frac{3}{4}\,\xi_0\right)$ near $\xi_0 = 0$. The time scale for the increase in the magnetic field due to differential rotation, equal to one over the rate of shear, is $m/2c$ in the local comoving proper time and $2m/3^{\frac{1}{2}}c\,\xi_0$ in the time of a distant observer.

I have only mentioned a few of the consequences of the use of the Kerr metric with $a^2 \simeq m^2$ to describe the geometry of space-time in the vicinity of a collapsed object which is accreting matter. The detailed calculations on which the above statements are based will be published elsewhere. It should be clear, however, that use of the Schwarzschild metric to describe a collapsed object may well not even give a qualitatively correct picture of the energy input into galactic nuclei, in the context of Lynden-Bell's basic hypothesis.

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JAMES M. BARDEEN

Department of Astronomy,
University of Washington,
Seattle, Washington 98105.

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Redshift and the Size of Double Radio Sources

A RELATIONSHIP between redshift and the angular size of radio sources, particularly quasi-stellar sources, has been sought with limited success for some time. Miley[1], for example, observed 72 quasi-stellar sources at 408 MHz and found that those with flat spectra and large redshift had on the average the highest fringe visibility. Bash[2], however, concluded from observations of 234 radio sources at 2,695 MHz that while fringe visibility was related to spectra, no significant relation to redshift was evident.

It now seems possible to establish a more definite relationship between angular size and redshift by using published data on radio sources. A study has been made of 32 radio galaxies and 25 quasi-stellar sources which are predominantly double in structure. Several details of the distribution of linear size of the sources may be significant.