



A review of Bondi–Hoyle–Lyttleton accretion

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Abstract

If a point mass moves through a uniform gas cloud, at what rate does it accrete material? This is the question studied by Bondi, Hoyle and Lyttleton. This paper draws together the work performed in this area since the problem was first studied. Time has shown that, despite the simplifications made, Bondi, Hoyle and Lyttleton made quite accurate predictions for the accretion rate. Bondi–Hoyle–Lyttleton accretion has found application in many fields of astronomy, and these are also discussed.

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1. Introduction

In its purest form, Bondi–Hoyle–Lyttleton accretion concerns the supersonic motion of a point mass through a gas cloud. The cloud is assumed to be free of self-gravity, and to be uniform at infinity. Gravity focuses material behind the point mass, which can then accrete some of the gas. This problem has found applications in many areas of astronomy, and this paper is an attempt to address the lack of a general review of the subject.

I start with a short summary of the original work of Bondi, Hoyle and Lyttleton, followed by a discussion of the numerical simulations performed. Some issues in Bondi–Hoyle–Lyttleton accretion are discussed, before a brief summary of the fields in which the geometry has proved useful.

2. Basics

This section is somewhat pedagogical in nature, containing a brief summary of the work of Bondi, Hoyle and Lyttleton. Readers familiar with the basic nature of Bondi–Hoyle–Lyttleton accretion may wish to skip this section.

2.1. The analysis of Hoyle and Lyttleton

Hoyle and Lyttleton (1939) considered accretion by a star moving at a steady speed through an infinite gas cloud. The gravity of the star focuses the flow into a wake which it then accretes. The geometry is sketched in Fig. 1.

Hoyle and Lyttleton derived the accretion rate in the following manner: Consider a streamline with impact parameter ζ . If this follows a ballistic orbit (it will if pressure effects are negligible), then we can apply conventional orbit theory. We have

$$\ddot{r} - r\dot{\theta}^2 = -\frac{GM}{r^2}, \quad (1)$$

$$r^2\dot{\theta} = \zeta v_\infty \quad (2)$$

in the radial and polar directions, respectively. Note that the second equation expresses the conservation of angular momentum. Setting $h = \zeta v_\infty$ and making the usual substitution $u = r^{-1}$, we may rewrite the first equation as

$$\frac{d^2u}{d\theta^2} + u = \frac{GM}{h^2}. \quad (3)$$

The general solution is $u = A \cos \theta + B \sin \theta + C$ for arbitrary constants A , B and C . Substitution of

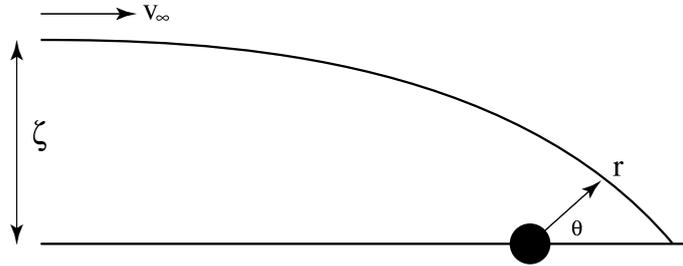


Fig. 1. Sketch of the Bondi–Hoyle–Lyttleton accretion geometry.

this general solution immediately shows that $C = GM/h^2$. The values of A and B are fixed by the boundary conditions that $u \rightarrow 0$ (that is, $r \rightarrow \infty$) as $\theta \rightarrow \pi$, and that

$$\dot{r} = -h \frac{du}{d\theta} \rightarrow -v_\infty \quad \text{as } \theta \rightarrow \pi.$$

These will be satisfied by

$$u = \frac{GM}{h^2} (1 + \cos \theta) - \frac{v_\infty}{h} \sin \theta. \quad (4)$$

Now consider when the flow encounters the $\theta = 0$ axis. As a first approximation, the θ velocity will go to zero at this point. The radial velocity will be v_∞ and the radius of the streamline will be given by

$$\frac{1}{r} = \frac{2GM}{h^2}. \quad (5)$$

Assuming that material will be accreted if it is bound to the star we have

$$\frac{1}{2} v_\infty^2 - \frac{GM}{r} < 0$$

or

$$\zeta < \zeta_{\text{HL}} = \frac{2GM}{v_\infty^2}, \quad (6)$$

which defines the critical impact parameter, known as the Hoyle–Lyttleton radius. Material with an impact parameter smaller than this value will be accreted. The mass flux is therefore

$$\dot{M}_{\text{HL}} = \pi \zeta_{\text{HL}}^2 v_\infty \rho_\infty = \frac{4\pi G^2 M^2 \rho_\infty}{v_\infty^3}, \quad (7)$$

which is known as the Hoyle–Lyttleton accretion rate.

2.2. Analytic solution

The Hoyle–Lyttleton analysis contains no fluid effects, which makes it ripe for analytic solution. This was performed by Bisnovatyi-Kogan et al. (1979), who derived the following solution for the flow field:

$$v_r = -\sqrt{v_\infty^2 + \frac{2GM}{r} - \frac{\zeta^2 v_\infty^2}{r^2}}, \quad (8)$$

$$v_\theta = \frac{\zeta v_\infty}{r}, \quad (9)$$

$$r = \frac{\zeta^2 v_\infty^2}{GM(1 + \cos \theta) + \zeta v_\infty^2 \sin \theta}, \quad (10)$$

$$\rho = \frac{\rho_\infty \zeta^2}{r \sin \theta (2\zeta - r \sin \theta)}. \quad (11)$$

The first three equations are fairly straightforward, and follow (albeit tediously) from the orbit solution given above. The equation for the density is rather less pleasant, and involves solving the steady state gas continuity equation under conditions of axial symmetry.

Eq. (4) may be rewritten into the form

$$r = \frac{r_0}{1 + e \cos(\theta - \theta_0)}, \quad (12)$$

where e is the eccentricity of the orbit, r_0 is the semi-latus rectum, and θ_0 is the periastron angle. These quantities may be expressed as

$$\theta_0 = \tan^{-1} \left(\frac{\zeta v_\infty^2}{GM} \right), \quad (13)$$

$$e = \sqrt{1 + \frac{\zeta^2 v_\infty^4}{G^2 M^2}}, \quad (14)$$

$$r_0 = \frac{\zeta^2 v_\infty^2}{GM}, \tag{15}$$

which may be useful as an alternative form to Eq. (10).

Note that these equations do not follow material down to the accretor. Accretion is assumed to occur through an infinitely thin, infinite density column on the $\theta = 0$ axis. This is not physically consistent with the ballistic assumption, since it would not be possible to radiate away the thermal energy released as the material loses its θ velocity. Even with a finite size for the accretion column, a significant trapping of thermal energy would still be expected. For now we shall neglect this effect.

2.3. The analysis of Bondi and Hoyle

Bondi and Hoyle (1944) extended the analysis to include the accretion column (the wake following the point mass on the $\theta = 0$ axis). We will now follow their reasoning, and show that this suggests that the accretion rate could be as little as half the value suggested in Eq. (7). Fig. 2 sketches the quantities we shall use.

From the orbit equations, we know that material encounters the $\theta = 0$ axis at

$$r = \frac{\zeta^2 v_\infty^2}{2GM}.$$

This means that the mass flux arriving in the distance r to $r + dr$ is given by

$$2\pi\zeta d\zeta\rho_\infty v_\infty = \frac{2\pi GM\rho_\infty}{v_\infty} dr = \Lambda dr, \tag{16}$$

which defines Λ . Note that it is independent of r . The transverse momentum flux in the same interval is given by

$$\Lambda v_\theta(\theta = 0) \frac{1}{2\pi s},$$

which is the mass flux, multiplied by the transverse velocity, divided over the approximate area of the wake. Applying the orbit equations once more, and noting that a momentum flux is the same as a pressure, we find

$$P_s \approx \frac{\Lambda}{2\pi s} \sqrt{\frac{2GM}{r}} \tag{17}$$

as an estimate of the pressure in the wake. The longitudinal pressure force is therefore

$$d(\pi s^2 P_s) = \Lambda \sqrt{\frac{GM}{2}} d\left(\frac{s}{\sqrt{r}}\right).$$

Material will take a time of about r/v_∞ to fall onto the accretor from the point it encounters the axis. This means that we can use the accretion rate to estimate the mass per unit length of the wake, m , as

$$m \approx \Lambda \frac{GM}{v_\infty^3}. \tag{18}$$

This makes the gravitational force per unit length

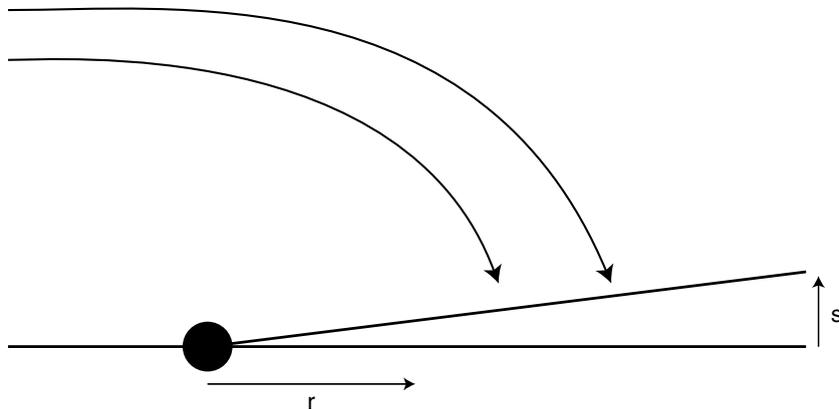


Fig. 2. Sketch of the geometry for the Bondi–Hoyle analysis.

$$F_{\text{grav}} = \frac{GMm \, dr}{r^2} \approx A \frac{G^2 M^2}{v_\infty^3} \frac{dr}{r^2}.$$

For accreting material, we must have $r \sim GMv_\infty^{-2}$. If we also assume that the wake is thin ($s \ll r$) and roughly conical ($ds/s \approx dr/r$), then taking the ratio of the pressure and gravitational forces, we find that pressure force is much less than the gravitational force. We can therefore neglect the gas pressure in the wake.

The mass per unit length of the wake, m , was introduced above. If we assume the mean velocity in the wake is v , we can write two conservation laws, for mass and momentum:

$$\frac{d}{dr}(mv) = A, \tag{19}$$

$$\frac{d}{dr}(mv^2) = Av_\infty - \frac{GMm}{r^2}. \tag{20}$$

Recall that Av_∞ is the momentum supply into the wake, since $\dot{r} = v_\infty$ on axis for all streamlines. We can declutter these equations by introducing dimensionless variables for m , r and v :

$$v = v_\infty v, \tag{21}$$

$$r = \frac{GM}{v_\infty^2} \chi, \tag{22}$$

$$m = \frac{AGM}{v_\infty^3} \mu. \tag{23}$$

Note that $\chi = 2$ corresponds to material arriving from the streamline characterised by ζ_{HL} . Substituting these definitions into Eqs. (19) and (20), we obtain

$$\frac{d}{d\chi}(\mu v) = 1, \tag{24}$$

$$\frac{d}{d\chi}(\mu v^2) = 1 - \mu \chi^{-2}. \tag{25}$$

We shall now analyse the behaviour of these equations.

We can integrate Eq. (24) to yield

$$\mu v = \chi - \alpha \tag{26}$$

for some constant α . Since μ is a scaled mass (and hence always positive), we see that the scaled velocity (v) changes sign when $\chi = \alpha$. That is, α is the stagnation point. Material for $\chi < \alpha$ will accrete,

so knowing α will tell us the accretion rate (since the accretion rate will be $A r_0$ where r_0 is the value of r corresponding to α). By writing $\mu v^2 = \mu v \cdot v$, we can use Eq. (26) to rewrite Eq. (25) as

$$v \frac{dv}{d\chi} = \frac{v(1-v)}{\chi - \alpha} - \frac{1}{\chi^2}. \tag{27}$$

This has not obviously improved matters, but we can now study the general behaviour of the function, without trying to solve it. First we need some boundary conditions. These are as follows:

- $v \rightarrow 1$ as $\chi \rightarrow \infty$
Which means that $v \rightarrow v_\infty$ at large radii
- $v = 0$ at $\chi = \alpha$
The stagnation point
- $\frac{dv}{d\chi} > 0$ Everywhere

The velocity must be a monotonic function. This is physically reasonable, if we are to avoid unusual flow patterns.

The first two conditions can be satisfied for any value of α . Fortunately, the third implies a restriction. The next set of manipulations may seem a little obscure at first, but they do lead in the desired direction.

Substitute $\xi = \alpha^{-1}\chi$. Eq. (27) then reads

$$v \frac{dv}{d\xi} = \frac{v(v-1)}{\xi-1} - \frac{1}{\alpha\xi^2}. \tag{28}$$

Now, suppose the derivative is zero. This leads to the condition

$$v^2 - v + \frac{1}{\alpha\xi^2}(x-1) = 0$$

or, one application of the quadratic roots formula later:

$$v = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{1}{\alpha\xi^2}(x-1)}. \tag{29}$$

Since v ultimately represents a physical quantity (the velocity), it is obviously desirable that it remain real. We therefore need to look at when the discriminant can become zero. This is another quadratic equation, leading to

$$\xi = \frac{2}{\alpha}(1 \pm \sqrt{1 - \alpha}),$$

which means that *something* must happen when $\alpha = 1$. To determine this ‘something,’ it is best to plot Eq. (29).

Figs. 3 and 4 demarcate the regions where $dv/d\xi$ changes sign, as dictated by Eq. (29). These are *not* possible solutions for v . However, any suitable solution for v must remain within the region marked ‘a’ if it is to remain monotonic and

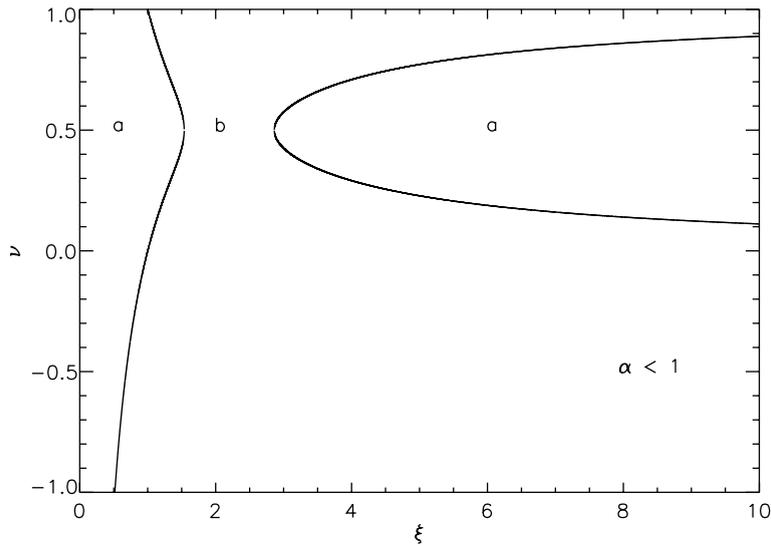


Fig. 3. Curves where $dv/d\xi = 0$ for $\alpha < 1$. In the regions marked ‘a,’ the derivative is greater than zero. It is less than zero in the ‘b’ region.

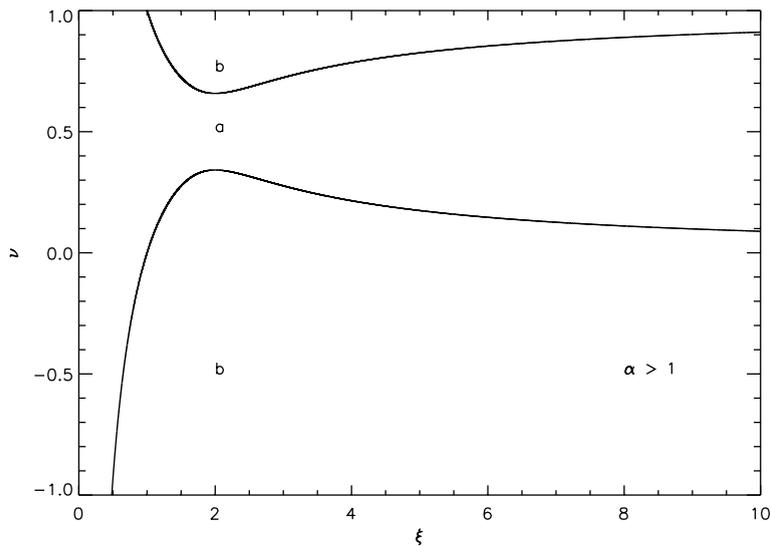


Fig. 4. Curves where $dv/d\xi = 0$ for $\alpha > 1$. In the region marked ‘a,’ the derivative is greater than zero. It is less than zero in the ‘b’ regions.

increasing. This is only possible when $\alpha > 1$. Unwinding our rescaled variables, we see that an α value of unity puts the stagnation point halfway between the accretor and the original value of Hoyle and Lyttleton. This in turn implies a minimum accretion rate of $0.5\dot{M}_{\text{HL}}$.

Again, I would like to remind the reader that the flow has been assumed to remain isothermal with negligible gas pressure throughout this discussion. This assumption is likely to be violated in the wake, where densities will be high and radiative heat loss inefficient. At the very least, thermal effects should be important close to the stagnation point in the wake. Horedt (2000) details an analysis similar to that given above, but with a pressure term included. The value of α (which Horedt calls x_0) was found to lie between 0.6 and 3.5 for flows which were supersonic at infinity and subject to Newtonian physics (the polytropic and adiabatic indices were also free parameters in this analysis).

Will the flow be stable? Bondi and Hoyle asserted that if $\alpha > 2$ (note that $\alpha = 2$ gives the solution of Hoyle and Lyttleton), then the wake would become unstable to perturbations which preserve axial symmetry. However, later analysis by Cowie (1977) suggested that the wake should be unstable, regardless of the value of α . Subsequent numerical simulations and analytic work have shown that Bondi–Hoyle–Lyttleton flow is far from stable, and we will discuss the subject in Section 4.2.

2.4. Connection to Bondi accretion

Bondi (1952) studied spherically symmetric accretion onto a point mass. The analysis shows (see e.g. Frank et al. (2002)) that a Bondi radius may be defined as

$$r_{\text{B}} = \frac{GM}{c_{\text{s}}^2(r_{\text{B}})}. \quad (30)$$

Flow outside this radius is subsonic and the density is almost uniform. Within it, the gas becomes supersonic and moves towards a freefall solution. The similarities between Eqs. (6) and (30) led Bondi to propose an interpolation formula:

$$\dot{M} = \frac{2\pi G^2 M^2 \rho_{\infty}}{(c_{\infty}^2 + v_{\infty}^2)^{3/2}}. \quad (31)$$

This is often known as the Bondi–Hoyle accretion rate. On the basis of their numerical calculations, Shima et al. (1985) suggest that Eq. (31) should acquire an extra factor of two, to become

$$\dot{M}_{\text{BH}} = \frac{4\pi G^2 M^2 \rho_{\infty}}{(c_{\infty}^2 + v_{\infty}^2)^{3/2}}, \quad (32)$$

which then matches the original Hoyle–Lyttleton rate as the sound speed becomes insignificant. The corresponding ζ_{BH} is formed by analogy with Eq. (7).

Nomenclature in this field can be a little confused. When papers refer to ‘Bondi–Hoyle accretion rates,’ they may mean Eqs. (7), (31) or (32). In this review, I shall refer to pressure-free flow as ‘Hoyle–Lyttleton’ accretion and use \dot{M}_{HL} and ζ_{HL} . When there is gas pressure, I will talk about ‘Bondi–Hoyle accretion’ and use \dot{M}_{BH} and ζ_{BH} , in the sense defined by Eq. (32). I shall use ‘Bondi–Hoyle–Lyttleton’ accretion to refer to the problem in general terms.

3. Numerical simulations

In the previous section, I outlined the basic theory behind Bondi–Hoyle–Lyttleton accretion. This led to elegant predictions for the accretion rate, as given by Eqs. (7) and (32). However, reaching these equations required a lot of simplifying assumptions, so necessitating further investigation. The intractability of the equations of fluid dynamics requires a numerical approach to the problem.

In a break with tradition, I shall start this section with the answer, and then give more detailed citations to examples.

3.1. Summary

Do the equations of Bisnovatyi-Kogan et al. provide a good description of Bondi–Hoyle–Lyttleton flow? The answer is ‘No.’

In the absence of other effects, three numbers parameterise Bondi–Hoyle–Lyttleton flow:

- The Mach number, \mathcal{M} .
- The size of the accretor, in units of ζ_{HL} .
- The γ value of the gas.

Fig. 5 shows sample density contours for a flow with $\mathcal{M} = 1.4$, an accretor radius of $0.1\zeta_{\text{HL}}$ and $\gamma = 5/3$. This particular simulation was axisymmetric. A bow shock has formed on the upstream side. The corresponding velocity field is plotted in Fig. 6. Downstream of the shock, material flows almost radially onto the accretor, in marked contrast to the analytic solution of Eqs. (8)–(11).

But what of the accretion rate? Fig. 7 shows the accretion rates obtained for three simulations. Although the dimensionless parameters were kept the same, the physical scales and grid resolution varied. Figs. 5 and 6 were taken from run 2. The

accretion rates achieved are quite close to the value of \dot{M}_{HL} predicted for the flow (this value is substantially larger than the corresponding \dot{M}_{BH}). Despite the simplifications made, the work of Bondi, Hoyle and Lyttleton has been largely vindicated. In the remainder of the section, I shall cite places in the literature where further simulations of Bondi–Hoyle–Lyttleton flow may be found.

3.2. Examples in the literature

Hunt (1971, 1979) computed numerical solutions of Bondi–Hoyle–Lyttleton flow. The accre-

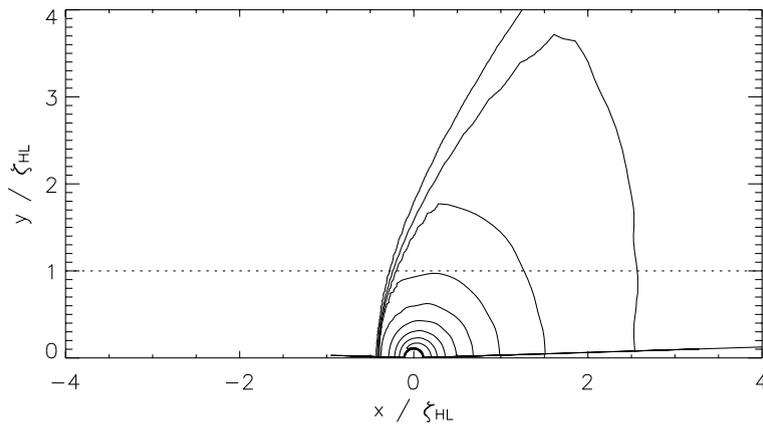


Fig. 5. Density contours for a sample Bondi–Hoyle–Lyttleton simulation. The flow had $\mathcal{M} = 1.4$, an accretor radius of $0.1\zeta_{\text{HL}}$ and the equation of state was adiabatic with $\gamma = 5/3$. The contours are logarithmically spaced over a decade of density. The dotted line indicates ζ_{HL} . The flow is incident from the left.

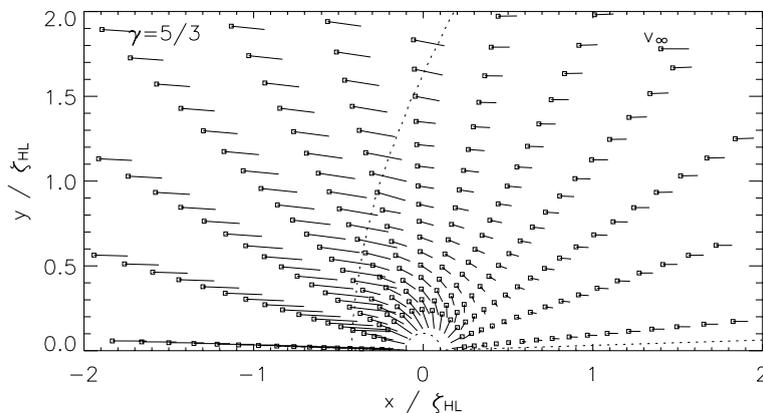


Fig. 6. Velocity field corresponding to the densities shown in Fig. 5. The approximate position of the bow shock is marked with a dotted line.

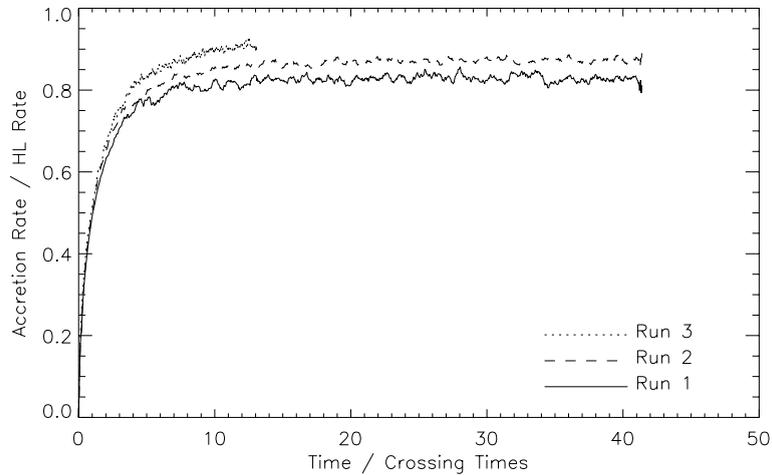


Fig. 7. Accretion rates for plain Bondi–Hoyle–Lyttleton flow. The crossing time corresponds to ζ_{HL} .

tion rate suggested by Eq. (32) agreed well with that observed, despite the flow pattern being rather different. Hunt studied flows which were not very supersonic and were non-isothermal. A bow shock formed upstream of the accretor. Upstream of the shock, the flow pattern was very close to the original ballistic approximation. Downstream, the gas flowed almost radially towards the point mass. A summary of early calculations of Bondi–Hoyle–Lyttleton flow may be found in Shima et al. (1985). The calculations in this paper are in broad agreement with earlier work, but some differences are noted and attributed to resolution differences.

More recently, a series of calculations in three dimensions have been performed by Ruffert in a series of papers (Ruffert, 1994a,b, 1995, 1996; Ruffert and Arnett, 1994). This series of papers used a code based on nested grids, to permit high resolution at minimal computational cost. Ruffert (1994a) details the code, and presents simulations of Bondi accretion (where the accretor is stationary). Bondi–Hoyle–Lyttleton flow was considered in Ruffert and Arnett (1994). The flow of gas with $\mathcal{M} = 3$ and $\gamma = 5/3$ past an accretor of varying sizes ($0.01 < r/\zeta_{\text{BH}} < 10$) was studied. For accretors substantially smaller than ζ_{BH} , the accretion rates obtained were in broad agreement with theoretical predictions. The flow was found

to have quiescent and active phases, with smaller accretors giving larger fluctuations. However, these fluctuations were far less violent than the ‘flip-flop’ instability observed in 2D simulations (see below). Ruffert (1994b) extended these simulations to cover a range of Mach numbers, finding that higher Mach numbers tended to give lower accretion rates (down to the original interpolation formula of Eq. (31)). Ruffert studied the flow of a gas with $\gamma = 4/3$ in the 1995 paper, finding accretion rates comparable with the theoretical results. Small accretors and fast flows were required before any instabilities appeared in the flow. Nearly isothermal flow was considered in Ruffert (1996). The accretion rates were slightly higher than the theoretical values (except for the smaller accretors), and the shock moved back to become a tail shock. The oscillations in the flow were less violent still.

The reason for the formation of the bow shock is straightforward – the rising pressure in the flow. As shown by Eq. (11), the flow is compressed as it approaches the accretor. This compression will increase the internal pressure of the flow, eventually causing a significant disruption. At this point, the shock will form. This interpretation is consistent with the behaviour observed in simulations, where decreasing γ moves the shock back towards the accretor. However, the precise

location of the shock does not seem to be a strong function of the Mach number (cf. the papers of Ruffert).

4. Issues in Bondi–Hoyle–Lyttleton flow

In this section I shall discuss some issues relating to Bondi–Hoyle–Lyttleton flow which are of particular interest.

4.1. The drag force

The simple idealisation of Bondi–Hoyle–Lyttleton flow cannot persist for long. The accretor is not only increasing its mass – it is accumulating momentum as well. Eventually, it should be accelerated to being co-moving with the gas flow. A full calculation is not straightforward, but dimensional considerations suggest

$$\dot{M}v_\infty \sim M\dot{v}_\infty = F_{\text{drag}}. \quad (33)$$

Please note, that the \sim in this equation is *very* approximate. However, Eq. (33) suggests that the accreting body will be brought to rest with respect to the flow on the mass doubling timescale. This is obviously a problem if the accretor is to change its mass appreciably. As we shall see later, this has led to most research into Bondi–Hoyle–Lyttleton accretion being concentrated into the study of binaries. In such cases, the momentum difference can be ‘paid’ by a change in orbit.

The drag does not originate as a form of ‘wind’ resistance pressing directly on the accretor. This is for two reasons.

- The momentum deposited by the accretion column will be far larger.
- Mathematically, the accretor is a point anyway.

Instead, the drag arises from the gravitational focusing of material behind the accretor. Since more material is present on the downstream side, the gravitational attraction of the downstream side is larger,¹ and exerts a drag force. Chandrasekhar (1943) was the first to consider this problem –

called ‘dynamical friction’ – for a collisionless fluid. An extended (and more recent) discussion of the problem is given by Binney and Tremaine (1987). Dokuchaev (1964) discussed the problem for a gaseous medium. The matter of drag is also mentioned by Ruderman and Spiegel (1971), who propose

$$F_{\text{drag}} = \dot{M}v_\infty \ln \left(\frac{b_{\text{max}}}{b_{\text{min}}} \right), \quad (34)$$

where b_{min} and b_{max} are cut-off radii for the gravitational force. Yabushita (1978) suggested that a suitable value for the outer cut-off for a flow with pressure would be the point where the pressure in the wake became equal to the background pressure (the inner cut-off radius is usually taken to be the radius of the accretor itself).

Values for the drag force given by Shima et al. (1985); Shankar et al. (1993); Ruffert and Arnett (1994) suggest that the drag force is no more than a factor of ten larger than the crude estimate of Eq. (33). The precise drag value has a tendency to fluctuate anyway – Bondi–Hoyle–Lyttleton flow is not stable.

4.2. Flow stability

Even in the axisymmetric case, there is no particular reason to believe that Bondi–Hoyle–Lyttleton flow should be stable. The binding energy test of Eq. (6) is made for gas flowing *away* from the accretor. If this material is going to be accreted, it needs to turn around somehow and fall towards the point mass. This must happen in some sort of accretion column, of the type first considered by Bondi and Hoyle (1944). As noted above, the work of Cowie (1977) found that this wake should be unstable. A ‘shock cone’ must surround the wake, in which the flow loses its θ velocity before it encounters the axis (see also Wolfson, 1977a,b). The high densities expected for the wake mean that this shock is likely to heat the gas. Gas pressure could then be expected to drive oscillations close to the stagnation point. Bondi–Hoyle–Lyttleton flow around small accretors has been studied by Koide et al. (1991). This paper notes that the ‘accreting body is so small that a part of the accreting gas sometimes misses the target

¹ This is ignoring the mathematical impossibility of an infinite, uniform medium for the unperturbed case.

object and flows towards the upstream as a jet.’ This is obviously a rather unstable condition, and leads to the accretion flow ‘sloshing’ back and forth around the point mass. The accretion rate fluctuates too, although the time averaged rate is still close to the Bondi–Hoyle value.

When the condition of axisymmetry is relaxed, even more instabilities become possible. Matsuda et al. (1987); Fryxell and Taam (1988); Taam and Fryxell (1988) performed 2D simulations of the Bondi–Hoyle–Lyttleton geometry, with the condition of axisymmetry relaxed, and a density and/or velocity gradient imposed on the upstream flow. All three papers found that a ‘flip-flop’ instability resulted, with the wake oscillating back and forth in a manner reminiscent of a von Kármán vortex street. Matsuda et al. (1991) suggested that the instability was intrinsic to the accretion flow, since it was found to develop even under the conditions first considered by Hoyle and Lyttleton (1939). Another detailed study of the ‘flip-flop’ instability for the 2D case for isothermal gas is that of Shima et al. (1998). The code used was specifically designed to conserve angular momentum and to permit very high resolution in the inner portions of the grid. They suggest that some of the resolution dependence of the instability found by earlier work was due to the use of codes which conserved *linear* momentum, and caution against the results obtained by such codes.

Such a spectacular instability naturally prompted an intense theoretical investigation. Soker (1990) extended the earlier work of Cowie (1977) to include tangential oscillations. The analysis is based on the assumption of a pressure free flow, and the expressions derived for the tangential behaviour also require the flow to be 2D and planar. The radial instability noted by Cowie was found to be independent of the incoming material. Any radial oscillation in the wake would grow, although the growth timescale was much longer than the oscillation timescales. The tangential modes (corresponding to the ‘flip-flop’ instability) behaved in a similar manner. Soker also predicted that the instability should be milder in the 3D case. In Soker (1991), a numerical study of the coupling between the radial and tangential oscillations was made. The radial

modes, corresponding to large density and velocity fluctuations were excited far beyond the linear regime, while the tangential oscillations remained linear. While the mass accretion rate showed corresponding fluctuations, the time averaged accretion rate was similar (although smaller) than the prediction of Eq. (7). Livio et al. (1991) added a simple analysis of the shock cone surrounding the wake seen in numerical simulations. Instabilities were found in both the planar 2D and full 3D cases, although the authors note that the instability should be milder in the 3D case (a point also made by Soker (1990)).

The major weakness of all simulations of the ‘flip-flop’ instability mentioned so far is that they fundamentally change the geometry of the problem. In order to simulate non-axisymmetric flow in 2Ds, the flow has to be assumed to be planar. This changes the shape of the accretor from a sphere to a cylinder. The equations of fluid dynamics are non-linear, and are notorious for their resolute refusal to yield to a proof of solution uniqueness (Fefferman, 2000). There is therefore no particular reason to expect the 2D planar simulations to be characteristic of the true solution in 3D. The simulations of Ruffert suggest that the ‘flip-flop’ instability is an artifact of 2D planar flow.

Foglizzo and Ruffert (1997, 1999) attempted to model the instabilities observed in the earlier numerical work of Ruffert. The first of these papers constructs stationary models, while the second contains a stability analysis. The origin of the instability was the bow shock generally seen in numerical simulations. This produces entropy gradients in the flow, which allows Rayleigh–Taylor and Kelvin–Helmholtz instabilities to grow. Foglizzo and Ruffert concluded that the instability should be stronger if

- The shock is detached from the accretor (as is the case for higher γ values).
- The flow has a higher Mach number.
- The accretor is smaller.

They found that the instability should be non-axisymmetric, and start at around $\theta = \pi/2$ and close to the accretor. Foglizzo and Tagger (2000) describe the instability as ‘entropic–acoustic,’

where entropy perturbations introduced by a shock propagate back to the shock via sound waves. These then trigger new entropy perturbations.

4.3. *Non-uniform boundary*

Suppose the conditions at infinity are not uniform, but instead a density and/or velocity gradient is present. This means that the flow within a cylinder of radius ζ_{HL} possesses angular momentum about the accretor. How much of this reaches the accretor?

Early calculations (Dodd and McCrea, 1952; Illarionov and Sunyaev, 1975; Shapiro and Lightman, 1976; Wang, 1981) suggested that most of this angular momentum would accrete. However, Davies and Pringle (1980) pointed out that only material which had lost most of its angular momentum would be able to settle onto a small accretor. They developed a simple analytic model for small density and velocity gradients (if the gradients become large, then the flow ceases to behave in the manner described by Bondi, Hoyle and Lyttleton). Davies and Pringle found that the mass accretion rate should be unaffected, and there should be no accretion of angular momentum.

Such confusion calls for numerical work. Ruffert and Anzer (1995) presented a sample 3D simulation of accretion under such conditions. They found that about 70% of the angular momentum available (as calculated by Shapiro and Lightman (1976)) would be accreted. Ruffert (1997) extended this calculation with papers in 1997 (covering velocity gradients) and Ruffert (1999) (studying density gradients). The mass accretion was not affected much, while the angular momentum accretion rate varied between 0% and 70% of the value suggested by Shapiro and Lightman (1976). Smaller accretors gave less stable flow, but none were as violently unstable as the ‘flip-flop’ instability observed in 2D planar simulations. Ruffert noted that a very small accretor will not be able to accrete all of the angular momentum within ζ_{HL} , but was unable to test such a case, due to the vast computational load involved in such a simulation.

4.4. *Radiation pressure*

Where there is accretion, there will be an accretion luminosity.² Radiative feedback has the potential to alter the Bondi–Hoyle–Lyttleton flow, and a number of workers have studied this.

Most work has concentrated on the problem of radiative feedback in X-ray binaries. This is a fairly straightforward application, since the constancy of the Thompson cross section makes the transfer problem intrinsically grey.

Blondin et al. (1990) simulated a compact object accreting an O star wind (forming an X-ray binary). Radiative heating was included (it was relevant to the radiation-driven wind), but the radiation force of the X-rays was neglected. The gas was assumed to be optically thin and in ionisation equilibrium. The wake was found to be unstable, oscillating back and forth. Despite these, the accretion rates were broadly consistent with that expected from a Bondi–Hoyle type analysis (some modifications were necessary to allow for the geometry of the binary). A later study of the same problem was made by Taam et al. (1991). This work included the effect of radiation pressure, but the flow was still assumed to be optically thin to electron scattering. Radiation pressure was then negligible except in the wake. This paper contained short section considering the accretion of an optically thin gas subject to radiation pressure. This led to the prediction that the flow would be unstable to oscillations if the accretion rate exceeded one third of the Eddington Limit.

Kley et al. (1995) were interested in the application of Bondi–Hoyle–Lyttleton accretion following a nova explosion in a binary. The radiation field was simulated using flux limited diffusion, and analytic approximations to opacity values. Their simulations had quite a complicated model for the accretor, including an envelope. Radiation pressure was found to be critical to simulating the flow (heating was included in all calculations, but the radiation pressure was omitted from some). For hot, optically thick flow, including radiation pressure made the flow subsonic and substantially

² Barring certain cases of finely tuned accretion onto a black hole.

reduced the drag. However, the accretion rate was found to be low (much less than the predictions of Bondi–Hoyle theory) in all cases.

4.5. Relativity

If the accreting object is a neutron star or a black hole, relativistic effects are likely to become important. However, relativistic hydrodynamics is generally recognised to be a non-trivial problem, and relatively little work has been done on Bondi–Hoyle–Lyttleton accretion for relativistic flows.

An early study by Petrich et al. (1989) found broad agreement between the Newtonian and relativistic cases. Font and Ibáñez (1998b) performed axisymmetric calculations in a Schwarzschild metric. The passage of time gave more powerful computers, enabling simulations to be run with higher resolution than Petrich et al. Most of the accretion rates found were similar to the Newtonian estimates, but some were an order of magnitude or so higher. No signs of instability were found. Font and Ibáñez (1998a) relaxed the assumption of axisymmetry, but still found the flow to be steady. However, the authors note that they were unable to push the simulation parameters very far.

Font et al. (1999) simulated Bondi–Hoyle–Lyttleton flow onto a rotating (Kerr) black hole. Two forms of the metric were used, to differentiate between numerical and physical effects. They found that rotational effects were confined to a region close to the hole. Their flows remained steady.

5. Applications

The Bondi–Hoyle–Lyttleton scenario has been applied to a variety of problems in the years since its introduction. I will now discuss some examples of these. Each of these problems could be the subject of a review article by themselves (and often have been), so I am not able to discuss the issues involved in great depth. However, I hope that this will provide a reasonable sample of the wide range of areas where the Bondi–Hoyle–Lyttleton model has proven useful.

5.1. Binary systems

The problem of accretion in a binary system seems to be the most popular application of the Bondi–Hoyle–Lyttleton analysis. As noted above, the inevitable drag force simply causes the orbit to change. The gas supply can either be from a stellar wind, or from common envelope (CE) evolution.

Wind accretion seems to be one of the most popular applications of the Bondi–Hoyle–Lyttleton geometry. However, there are a number of potential complications. For the Bondi–Hoyle–Lyttleton solution to be valid, accretion must be driven by a wind, rather than by Roche lobe overflow. The work of Petterson (1978) warns that the presence of Roche lobe overflow will substantially complicate matters, and that allowance must be made for the possibility when comparing theory with observations. The orbital motion of the binary can also cause problems – see Theuns and Jorissen (1993); Theuns et al. (1996) for a discussion. These papers studied accretion rate in a binary, where the wind speed was comparable to the orbital velocity. Consequently, the flow pattern was substantially different from that envisaged by Bondi, Hoyle and Lyttleton. Theuns et al. (1996) found that the accretion rate for a binary is decreased by a factor of about ten compared to the prediction of Eq. (32). They attribute this difference to the disrupting effect of the orbital motion. Moving to higher mass ratios, Struck et al. (2004) modelled wind accretion onto substellar companions of Mira variables. They found that the mean accretion rates were generally similar to those predicted by Bondi, Hoyle and Lyttleton. However, the flow was highly variable – both due to instabilities in the accretion flow and the intrinsic variability of the star. The ultimate ‘wind’ is that produced by an explosion. In this case, the very high velocities involved tend to make ζ_{BH} comparable to the size of the accreting body. The work of Kley et al. (1995) has already been mentioned. However, the problem had been studied before – MacDonald (1980) estimated accretion rates using modified a Bondi–Hoyle–Lyttleton formula.

Jackson (1975) used the predictions of Bondi–Hoyle–Lyttleton theory to derive the system parameters of Cen X-3. Similar work was performed

by Eadie et al. (1975), Pounds et al. (1975) and Lamers et al. (1976). By considering the accretion of a high velocity wind by neutron stars, Pfahl et al. (2002) concluded that most of the low luminosity, hard X-ray sources known in our Galaxy could be powered by such systems. Modified Bondi–Hoyle–Lyttleton accretion has also been used to study cases where a giant star’s wind is being accreted by a main sequence star. Some examples are the work of Chapman (1981) and Che-Bohnenstengel and Reimers (1986). The Bondi–Hoyle–Lyttleton geometry is a useful first approximation to wind accretion in binary systems. However, unless the wind speed is much greater than the orbital velocity, the accretion rates can deviate significantly from the simple predictions.

At the extreme end of the mass ratio scale, Bondi–Hoyle–Lyttleton accretion has even been used to estimate accretion rates onto a planet embedded in a disc (Nelson and Benz, 2003). Although the situation simulated was not entirely appropriate to the original analysis, Nelson and Benz point out that it represents a *maximum* possible accretion rate. This rate turns out to be extremely high, showing the need for higher resolution simulations of planetary accretion flows.

Common Envelope (CE) evolution occurs when two stellar cores become embedded in a large gas envelope. Such an envelope is typically produced when one of the members of the binary system swells as it leaves the main sequence. For a more detailed discussion of CE evolution itself see, e.g. Iben and Livio, 1993. In such cases, accretion rates are critical for determining the detailed evolution of the system. In computing accretion rates, modified Bondi–Hoyle–Lyttleton formulae are often used e.g. Taam and Bodenheimer, 1989 – see also the review by Taam and Sandquist (2000).

5.2. Protostellar clusters

Bondi–Hoyle–Lyttleton flow is also likely to be applicable to regions of star formation. Although single stars will be stopped by the drag force, real stars are generally thought to form in clusters. Protostars and gas are trapped inside a gravitational well, and orbit within it. The drag will then

simply cause a change in orbit – as is the case for X-ray binaries. Indeed, the approximation is likely to be better for protostars in a protocluster. This is because the orbital motion of the protostars is the ‘source’ of the wind, rather than a wind from a companion. As a result, the geometry is simpler (since the orbital motion does not have to be added to the wind velocity). Furthermore, non-inertial forces (coriolis and centrifugal) are likely to be far less significant.

Bonnell et al. (2001) performed a thorough study of accretion in a protocluster. They simulated the evolution of a gas cloud containing many small point masses (representing protostars). The point masses grew by accreting the gas. When the gas dominated the mass of the cluster, Bonnell et al. found that the accretion was best described by tidal lobe overflow (examining whether material was bound to the cluster or the star). However, as the mass in stars grew, Bondi–Hoyle–Lyttleton accretion became the more significant mechanism. The transition occurred first for the most massive stars which had sunk into the cluster core. However, massive stars are very luminous and Edgar and Clarke (2004) showed that radiative feedback can disrupt the Bondi–Hoyle–Lyttleton flow once stellar masses exceed $10M_{\odot}$ (this is obviously dependent on the prevailing conditions in the protocluster).

Unfortunately, direct observations of this process are not available. Protoclusters contain large quantities of dusty gas, which greatly obscure regions of interest. Furthermore, the expected luminosities are lower, and the emission wavelengths less distinctive than those of X-ray binaries.

5.3. Galaxy clusters

Galaxies orbiting in a cluster are another candidate for Bondi–Hoyle–Lyttleton accretion. One immediate complication is the high temperature of the intergalactic medium (IGM). The IGM is typically hot enough to emit X-rays, and hence the galactic motions will usually be subsonic. Ruderman and Spiegel (1971) suggested that the IGM might be heated (at least in part) by the accretion shocks inherent to Bondi–Hoyle–Lyttleton accretion. Galaxies are also rather porous objects, and

contain their own gas. In a study of M86, Rangarajan et al. (1995) concluded that the ‘plume’ observed was probably the result of ram-pressure stripping of material from the galaxy itself. Stevens et al. (1999) simulated galaxies under such conditions, and concluded that “the ram-pressure stripped tail will usually be the most visible feature.” This paper also contains a list of observed wakes.

De Young et al. (1980) observed M87, and found evidence for *subsonic* Bondi–Hoyle–Lyttleton flow. However, higher resolution observations by Owen et al. (2000) suggest that this simple picture is not sufficient. In particular, the active nucleus of M87 drives an outflow. A filament has been observed trailing Abell 1795 both in the optical (Cowie et al., 1983) and in X-rays (Fabian et al., 2001). It has been proposed (Sakelliou et al., 1996) that this filament is an accretion wake, but Fabian et al. (2001) note that the gas cooling times aren’t quite right for this simple approximation to be completely valid. Sakelliou (2000) constructed a simple theoretical model of a Bondi–Hoyle–Lyttleton wake behind a galaxy. Wakes were expected to extend for up to 20 kpc. The wakes would form behind slow moving, massive galaxies in low temperature clusters.

5.4. Other applications

The original application of Bondi–Hoyle–Lyttleton accretion was to the flow of the interstellar medium past the Sun. Sikivie and Wick (2002) apply a similar analysis to the flow of dark matter past the Sun. They suggest that annual variations in WIMP detections may be partially attributable to the focusing of the flow by the Sun.

Bondi–Hoyle–Lyttleton accretion was invoked by Kamp and Paunzen (2002) to explain the unusual chemical abundances of λ -Bootis type stars. These stars have metal abundances typical of the interstellar medium (that is, metal-poor). Kamp and Paunzen suggest that radiation pressure on dust grains (which contain most of the metals) prevents the accretion of the heavier elements, while gas accretes in a Bondi–Hoyle–Lyttleton fashion. Moving to higher energies, accretion onto neutron stars moving through gas clouds has been

proposed as a mechanism for producing X-ray sources in the Galaxy (Ostriker et al., 1970) and in globular clusters (Pfahl and Rappaport, 2001). However with neutron stars, magnetic fields can cause significant complications – see, e.g. Toropina et al., 2001.

Maeda et al. (2002) studied the central portions of our Galaxy with the Chandra X-ray observatory. Finding evidence for recent activity, they suggest that this could have been powered by the central black hole accreting material from an expanding supernova shock. This would be a transient example of Bondi–Hoyle–Lyttleton accretion. The potential luminosity from this is rather high (comparable with the Eddington Limit). However, there is a complication due to the thermal pressure of the ambient gas, which could reduce the inferred luminosity substantially.

6. Summary

In their original analyses, Bondi, Hoyle and Lyttleton made many simplifications. Despite these, the broad picture they present seems to be correct. Numerical studies have been made of the purely hydrodynamic problem, and of cases where extra physical processes are relevant. Bondi–Hoyle–Lyttleton accretion has also been used to explain phenomena in a variety of astronomical contexts.

Of course, the original Bondi–Hoyle–Lyttleton results cannot be applied without some thought. Numerical studies have shown that the flow pattern is more complicated than that originally envisaged. Meanwhile, real systems are always more complicated than theoretical ones. Bondi–Hoyle–Lyttleton accretion should be regarded as a reference model – it is unlikely to explain any system in detail, but it can serve as a useful basis for classifying behaviours. It can be applied as a test model on systems of all scales – from binary stars up to galaxies in clusters.

There are many future avenues for research. As well as improving simulations of accretion in binaries, studies need to be made on Bondi–Hoyle–Lyttleton accretion for flows where the accretor is orders of magnitude smaller than the accretion

radius. Previous numerical work has hinted at lurking instabilities, but sufficient computing resources have yet to be brought to bear. Extra physical processes are also candidates for inclusion, especially radiative feedback. Radiative feedback is likely to be relevant in the context of both X-ray binaries and star formation simulations, and could alter the flow pattern significantly. On the observational side, better observations of systems previously modelled with a simple Bondi–Hoyle–Lyttleton analysis will show deviations, which can then be used to enhance our understanding of them.

Time has not eroded the usefulness of the Bondi–Hoyle–Lyttleton accretion geometry. It provides a simple framework for examining and refining theory and observation.

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