



Revised Upper Limit to Energy Extraction from a Kerr Black Hole

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We present a new upper limit on the energy that may be extracted from a Kerr black hole by means of particle collisions in the ergosphere (i.e., the “collisional Penrose process”). Earlier work on this subject has focused largely on particles with critical values of angular momentum falling into an extremal Kerr black hole from infinity and colliding just outside the horizon. While these collisions are able to reach arbitrarily high center-of-mass energies, it is very difficult for the reaction products to escape back to infinity, effectively limiting the peak efficiency of such a process to roughly 130%. When we allow one of the initial particles to have impact parameter $b > 2M$, and thus not get captured by the horizon, it is able to collide along outgoing trajectories, greatly increasing the chance that the products can escape. For equal-mass particles annihilating to photons, we find a greatly increased peak energy of $E_{\text{out}} \approx 6 \times E_{\text{in}}$. For Compton scattering, the efficiency can go even higher, with $E_{\text{out}} \approx 14 \times E_{\text{in}}$, and for repeated scattering events, photons can both be produced *and* escape to infinity with Planck-scale energies.

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When two particles scatter or annihilate close to a spinning black hole, it is possible that one of the products can have negative energy, necessarily getting captured by the horizon, thereby reducing the black hole’s total mass. This energy is transferred to the other particle, which then may escape to infinity with more energy than that of the initial particles. This process, first identified by Penrose in the context of decay products, has long been studied as a curious property of Kerr black holes, but is of questionable astrophysical relevance [1].

Notably, early work by Wald [2] showed that a maximum efficiency of $\approx 121\%$ can be obtained for a single particle falling from rest at infinity, then decaying into two equal-mass particles in the ergosphere, one of which escapes, and one of which is captured. Piran et al. found qualitatively similar results for a variety of scattering and annihilation processes, with only modest net energy extraction from the black hole despite arbitrarily large center-of-mass energies [3,4].

More recently, the subject has received renewed interest following a paper by Banados, Silk, and West (BSW) [5] that proposed to use extremal Kerr black holes to probe Planck-scale energies by colliding particles on critical trajectories just outside the horizon. Numerous subsequent papers pointed out that, while very large center-of-mass energies are indeed obtainable, the subsequent redshift of the escaping particles ensures that the net efficiency of the Penrose process remains limited to only about 130% [6–10].

One feature that all these papers have in common is their analytic approach to the problem. Based on strong symmetry arguments, they focus almost exclusively on very specific geodesic trajectories in the equatorial plane,

usually taking the limit as the point of collision approaches the horizon, where the center-of-mass energy is highest. In a companion work [11], we have taken a different approach, calculating numerically the distribution function of dark matter particles around a Kerr black hole using a fully three-dimensional Monte Carlo code to integrate a huge sample of random geodesic orbits for both particles and annihilation photons [12]. In doing so, we consistently found a small number of photons that managed to escape the black hole with energies well in excess of the theoretical bounds of Refs. [8,9].

In this Letter, we focus on these extreme events, again returning to analytic methods to understand the properties of the colliding particles, their products, and the dependence on physical parameters such as the black hole spin. We arrive at the somewhat startling conclusion that, for annihilation between equal-mass particles falling from rest at infinity, the escaping photons can attain energies equal to *six times* the total rest mass energy of the incoming particles. For Compton scattering between a photon and massive particle, the efficiency can reach nearly 1400%! Finally, we show how repeated scattering events can increase the energy of a photon—and the net efficiency of the Penrose process—almost without bound.

We begin with a description of geodesic trajectories in the equatorial plane around a Kerr black hole. From the normalization constraint $g_{\mu\nu}p^\mu p^\nu = -m^2$, we can write down an effective potential,

$$V_{\text{eff}}(r) = k \frac{M}{r} + \frac{\ell^2}{2r^2} + \frac{1}{2}(-k - \varepsilon^2) \left(1 + \frac{a^2}{r^2} \right) - \frac{M}{r^3} (\ell - a\varepsilon)^2, \quad (1)$$

where r is the radius in Boyer-Lindquist coordinates, ℓ and ε are the particle's specific angular momentum and energy, M and a are the black hole mass and spin, and $k = 0$ for photons and $k = -1$ for massive particles. We set $G = M = c = 1$ throughout this Letter. For a specific choice of a , ℓ , ε , and k , we can solve for the radial turning points by setting $V_{\text{eff}}(r) = 0$.

Figure 1 shows these turning points as a function of the impact parameter $b \equiv \ell/\varepsilon$ for both massless and massive particles, for maximal spin $a = 1$. For the massive particles, we set $\varepsilon = 1$, corresponding to a particle at rest at infinity. One can imagine a massive particle incoming from the right with $b < -2(1 + \sqrt{2})$ or $b > 2$, reflecting off the centrifugal potential barrier and returning back to infinity (yellow regions). Alternatively, if the impact parameter is small enough [i.e., $-2(1 + \sqrt{2}) < b < 2$], the particle will get captured by the black hole. Due to frame dragging, the cross section for capture is much greater for incoming particles with negative angular momentum.

For a given $\ell = p_\phi$ and $\varepsilon = -p_r$, the radial momentum p_r can be determined from the normalization condition $p_\mu p_\nu g^{\mu\nu} = k$,

$$p_r = \pm [g_{rr}(k - g^t \varepsilon^2 + 2g^{\phi} \ell \varepsilon - g^{\phi\phi} \ell^2)]^{1/2}, \quad (2)$$

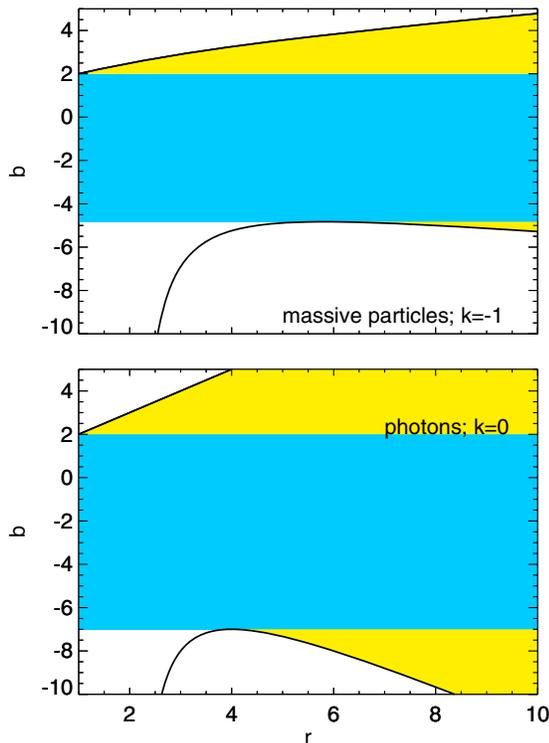


FIG. 1 (color online). Radial turning points in the effective potential $V_{\text{eff}}(r, b)$ for (top) massive and (bottom) massless particles, for a black hole with maximal spin $a = 1$. Any particle in the yellow region can escape from the black hole, but in the blue regions, only particles with outgoing radial velocities can escape. The static limit is located at $r = 2$ and the horizon is at $r = 1$.

and the sign of the root is chosen depending on the criterion described below.

Let us now consider a simple reaction between two particles with momenta $\mathbf{p}^{(1)}$ and $\mathbf{p}^{(2)}$ that collide to produce two particles with $\mathbf{p}^{(3)}$ and $\mathbf{p}^{(4)}$. Constraining all trajectories to the equatorial plane, we have to solve for six unknown momentum components. Conservation of total momentum provides three constraints, and the normalization conditions on the two daughter particles provide two more. We are left with a single free parameter: the angle ψ between $\mathbf{p}^{(3)}$ and the $\hat{\mathbf{r}}$ direction in the center-of-mass frame.

After solving for the momenta $\mathbf{p}^{(3)}$ and $\mathbf{p}^{(4)}$ of the products, we determine if they can escape the horizon based on the effective potential barriers and the sign of the radial velocity component, as evident in Fig. 1. Scanning over all values of the angle ψ , we solve for the maximum energy of escaping particles as a function of radius. Then the peak efficiency of the reaction is defined as $\eta_{\text{max}} = \varepsilon_{\text{max}}^{(3)}/(\varepsilon^{(1)} + \varepsilon^{(2)})$. Interestingly, as pointed out by Refs. [4,9], the highest energy photons that can escape are initially ingoing with $b_3 > 2$, giving $\eta_{\text{max}} \approx 1.3$, while those photons with initially outward-directed trajectories typically have $\eta_{\text{max}} < 0.5$.

BSW and much of the subsequent work on this topic focus predominantly on reactions between equal-mass particles falling from rest at infinity with critical values of the impact parameters ($b_1 = 2$, $-2(1 + \sqrt{2}) \leq b_2 \leq 2$), which lead to divergent center-of-mass energies just outside the horizon when $a = 1$. Yet the products of such reactions are either immediately captured by the black hole, or lose most of their energy due to gravitational redshift before reaching an observer [7–10].

In Fig. 2, we plot $\eta_{\text{max}}(r)$ for the same parameters as in Fig. 2(b) of Ref. [9], and reassuringly get identical results. In all cases, $m_1 = m_2 = 1$, $m_3 = m_4 = 0$, $\varepsilon_1 = \varepsilon_2 = 1$, $a = 1$, $b_1 = 2$, and $p_r^{(1)}, p_r^{(2)} < 0$. One interesting result is

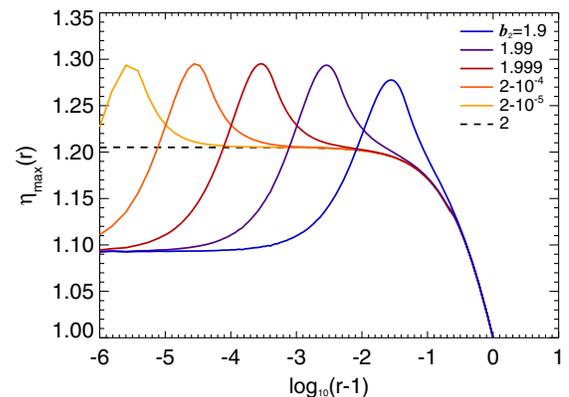


FIG. 2 (color online). Peak efficiency for annihilation of equal-mass particles falling from rest at infinity, as a function of the radius at which the annihilation occurs. The impact parameter $b_1 = 2$ is fixed at the critical value, and b_2 varies. The black hole spin is maximal: $a = 1$. (Compare to Fig. 2 of Ref. [9])

that, while the center-of-mass energy increases with smaller r , the peak efficiency is reached somewhat outside the horizon, due to the greater probability of escape. Note also that the efficiency can only surpass unity for $r < 2$, because the Penrose process only works inside the ergosphere. The dashed line, for $b_2 = b_1 = 2$, is physically equivalent to spontaneous breakup of a single particle into two photons, as in Ref. [2].

While exploring the distribution function and annihilation products of dark matter around Kerr black holes [11], we consistently found escaping photons with energies greater than the limits of Bejger et al. [9]. An important clue to understanding these seemingly impossible photons came from Fig. 2: the greatest efficiency actually comes from trajectories somewhat outside the horizon. When considering initial particles with $b > 2$ that never reach the horizon, it soon became obvious that they can reflect off the effective potential barrier and escape with $p_r > 0$. When colliding with particles on incoming trajectories, these “supercritical” particles contribute some positive radial momentum to the center-of-mass frame, thereby increasing the range of angles—as measured in the coordinate frame—accessible to the product photons. This in turn greatly increases the chance that high-energy products can escape to infinity (even if they are initially on ingoing trajectories).

In Fig. 3, we again plot the peak efficiency for annihilation reactions between two equal-mass particles with $\varepsilon_{1,2} = 1$. Yet now we set b_1 to be infinitesimally larger than the critical value, allowing $p_r^{(1)} > 0$. The difference is quite profound. (Mathematically, the difference must only be infinitesimally larger, e.g., 10^{-300} , in order to reflect off the potential barrier. Of course, this is not numerically resolvable by even quadruple precision, so in practice we set b_1 to the critical value.) Not only is the peak efficiency much greater than previously calculated, but it is also attainable for a wider range of b_2 values.

To better understand the cause of this increase, in Fig. 4 we plot the center-of-mass energy E_{com}/m as a function of radius, for both $p_r^{(1)} > 0$ (solid curves) and $p_r^{(1)} < 0$ (dashed curves). The values for b_2 and the color scheme are identical to Fig. 3. Interestingly, the peak center-of-mass energy is only a factor of ~ 2 higher for outgoing particles relative to ingoing particles. Thus we conclude that the huge increase in efficiency in our case is due to a higher escape probability for the products, rather than a much higher initial photon energy.

We also considered Compton-like scattering between a photon and massive particle in a similar geometry to the annihilation mechanism, with $b_1 = 2$, $p_r^{(1)} > 0$, $b_2 = -2(1 + \sqrt{2})$, and $p_r^{(2)} < 0$. The results are shown in Fig. 5, where the different curves correspond to different photon energies ε_1 , and the efficiency is still defined as $\varepsilon_3/(\varepsilon_1 + \varepsilon_2)$. (The “kinks” in the Fig. 5 curves correspond to different roots of the quadratic equation that arise when solving for the momentum constraints and escape

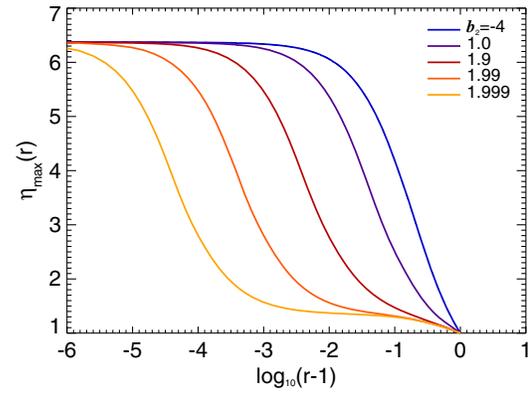


FIG. 3 (color online). Peak efficiency for annihilation of equal-mass particles falling from rest at infinity, as a function of the radius at which the annihilation occurs. Unlike in Fig. 2, here we allow $p_r^{(1)} > 0$, which greatly increases the fraction and energy of escaping photons. The impact parameter b_1 is fixed at 2 and b_2 varies.

conditions for $\mathbf{p}^{(3)}$.) When $\varepsilon_1 \gg m_2$, we find the escaping photons can reach energies roughly 14 times that of the incoming photons.

As noted above, the highest-energy photons that are able to escape are actually emitted with ingoing radial momentum before reflecting off the potential barrier and then reaching infinity. This allows for the fortunate possibility of multiple scattering events with the same photon: each time, the photon first reflects off the potential barrier so that $p_r^{(1)} > 0$, then scatters off a new infalling particle with $\varepsilon_2 = 1$, $b_2 = -2(1 + \sqrt{2})$, and the resulting photon has $p_r^{(3)} < 0$ and $b_3 > 2$, allowing the process to repeat.

Thus it is possible to reach photon energies on the order of $m_2 10^N$ after N scattering events, for a net efficiency of $\eta \approx 10^N/N$. The only limitation, as can be seen in Fig. 5, is that the radius at which such high-efficiency reactions

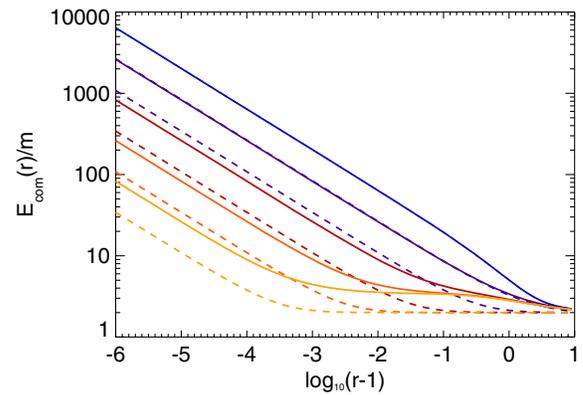


FIG. 4 (color online). Center-of-mass energy for the annihilation reactions shown in Figs. 2 and 3. The color scheme is as in Fig. 3, with solid lines corresponding to $p_r^{(1)} > 1$ and dashed lines for $p_r^{(1)} < 1$. The highest center-of-mass energies naturally occur for the prograde-retrograde collisions, which are closest to “head-on.”

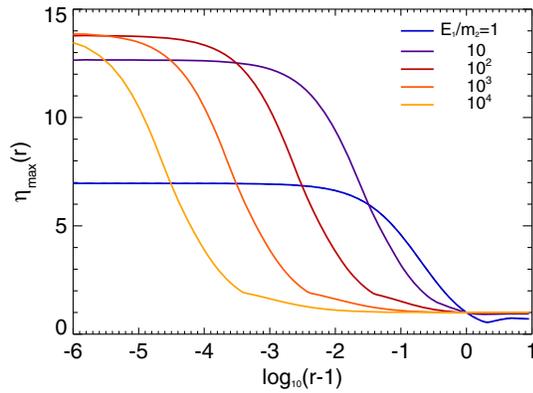


FIG. 5 (color online). Peak efficiency for Compton-like scattering events between a photon with $b_1 = 2$, $p_r^{(1)} > 1$ and a massive particle falling from rest at infinity with rest mass m_2 and impact parameter $b_2 = -2(1 + \sqrt{2})$.

($\eta \gtrsim 10$) can occur moves steadily inward with each subsequent scattering event. We call this radius $r_{\text{crit},N}$ and find that $\log_{10}(r_{\text{crit},N} - 1) \approx -1 - N$.

At the same time, each scattering event deposits some negative angular momentum into the black hole, reducing its spin. This in turn changes the effective potential, and increases both the impact parameter and turning radius for the critical trajectories required for high-efficiency scattering events. In the limit of $\epsilon \equiv (1 - a) \ll 1$, we find the critical value is $b \approx 2(1 + \epsilon^{1/2})$, which corresponds to a turning radius of $r_{\text{turn}} \approx 1 + 2\epsilon^{1/2}$. Eventually, the spin is reduced to a point where the turning radius is outside of r_{crit} and the efficiency begins to suffer (see Fig. 6).

Following Ref. [13], we can write an expression for the spin evolution due to capture of a particle with angular momentum δJ and energy δE ,

$$\delta a = -2a \frac{\delta E}{M} + \frac{\delta J}{M^2}. \quad (3)$$

Assuming each scattering event has an efficiency of $\eta = 10$, and we start with $a = 1$, the change in spin due to the N th event is to leading order

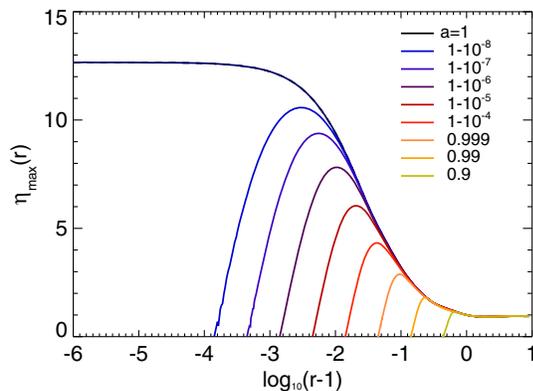


FIG. 6 (color online). Spin dependency for efficiency of the scattering events plotted in Fig. 5 with $\epsilon_1/m_2 = 10$.

$$\delta \epsilon_N \approx 2 \frac{m_2}{M} (2 + \sqrt{2} + \epsilon_N^{1/2} 10^{N+1}), \quad (4)$$

where $1 - \epsilon_N$ is the spin just before the N th event. (While each postscattering particle m_4 deposits a greater amount of negative angular momentum, it also deposits more negative energy, so the net change in spin is relatively constant.) Since the condition $r_{\text{turn}} < r_{\text{crit}}$ requires $\epsilon_N^{1/2} 10^{N+1} < 1/2$, we can write

$$\epsilon_{N+1} \approx (4 + 2\sqrt{2})N \frac{m_2}{M}, \quad (5)$$

which in turn gives the maximum number of high-efficiency scattering events: $N_{\text{max}} \approx \log_{10}(M/m_2)^{1/2}$.

Taking m_2 to be the electron mass and $M = 10M_{\odot}$ gives $N_{\text{max}} \approx 30$, for a peak energy of 10^{26} GeV. Thus photons undergoing repeated Compton scattering events could not only far surpass the Planck energy scale, but these hyper-energetic photons could even escape to an observer at infinity!

By simply relaxing a single assumption about the initial trajectories of colliding particles, we have identified a mechanism that allows high-energy particles to be produced by the collisional Penrose process and also escape to infinity. It is entirely possible that further exploration of parameter space will find even more extreme cases that have not yet been considered in this Letter.

At the same time, it is eminently clear that the most extreme effects will have little or no application to astrophysical processes, where random particle trajectories and realistic black hole spins will likely never reach the critical levels required for such events. For example, quantum effects and Hawking radiation will naturally spin down even an isolated extremal black hole [14], and gravitational radiation from the scattering particles could further degrade the spin [15] (although a carefully constructed steady-state stream of incoming particles might avoid this problem by removing the time-varying quadrupole moment). A more astrophysically realistic calculation of potentially observable signatures from dark matter annihilations is given in Ref. [11].

One might imagine an advanced civilization that can create extremal black holes and harness them as energy storage devices or particle accelerators. Of course, there is the problem of spinning down the black hole in the process, so that only a single extreme event might ever be achieved [15], but this too could be avoided by providing a simultaneous source of positive angular momentum via accretion. The net efficiency would suffer, but such is the cost of probing Planck-scale physics in the laboratory.

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