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HIGH EFFICIENCY OF THE PENROSE MECHANISM FOR PARTICLE COLLISIONS

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ABSTRACT

It is shown that, by two-particle collisions, rather than particle disintegration, one substantially increases the efficiency of the Penrose mechanism. The process may be relevant when considering both means of extracting energy from black holes and means of producing energetic particles. The process can be operating in close binary systems, such as some compact X-ray sources, of which one of the members is a black hole.

Subject headings: black holes — X-ray sources

When a test body A falls into a Kerr black hole (BH) and disintegrates in the ergosphere into two other test bodies B and C, it may happen that B (or C) will be ejected carrying energy in excess of that of the incoming particle A. This is the well-known Penrose process (Penrose 1969), by which energy can be extracted from a rotating BH. The energy gain by this process can be shown to have an upper bound, which is a certain fraction of the rest mass μ_A (Wald 1974; Kovetz and Piran 1974). Wheeler (1970) and others have speculated on the possibility that the breakup of a star by the tidal gravitational forces of a giant black hole could result in the extraction of energy from the latter by the Penrose process. However, as Bardeen, Press, and Teukolsky (1972) and Wald (1974) have pointed out, the high energy of the ejected body must come either from a high initial kinetic energy of A or from its breakup energy, not from the rotational energy of the black hole.

Disintegration, however, is not the only way by which the Penrose process may be realized. Consider, for instance, a four-body reaction in which two particles A and B collide inside the ergosphere to give $A + B \rightarrow C + D$. In this case, the total energy S in the center-of-mass frame (see below) replaces the rest mass μ_A (of the three-body Penrose process) in estimating upper bounds on energy gains; and since S can become very large near the horizon, energy will be extracted from the black hole far more efficiently. In this way one can produce particles which are very energetic indeed.

The four-body Penrose mechanism has, in fact, an important practical aspect. Recently, a great deal of research has been done on binary systems in which a compact member (possibly a black hole) is accreting matter from its companion, the accretion being accompanied by X-radiation. This Penrose process, involving p-p collisions or the Compton effect, could, with a suitable geometrical distribution of matter, provide an additional source of energy in binaries with a black hole; as the process can occur in a non-steady–state fashion inside the last stable orbit, one could extract energy in excess of its binding energy.

We shall describe a Kerr black hole of mass M and angular momentum per unit mass a in the familiar Boyer-Lindquist (1967) metric which, in coordinates $(X^\lambda) \equiv (x^0, x^k \equiv (r, \theta, \phi))$ has the form

$$ds^2 = e^{2\nu}(dx^0)^2 - e^{2\psi}(d\phi - \omega dx^0)^2 - e^{2\mu_1}(dr)^2 - e^{2\mu_2}(d\theta)^2 . \qquad (1)$$

The functional dependence of ν, ψ, ω, μ_1, and μ_2 on r, θ, ϕ, M, and a may be found in the original reference; since we shall not make explicit use of it here, we shall not copy it down. We shall mainly be concerned with the fact that, on approaching the horizon, $e^{-\nu}$ goes to infinity while ωe^{ψ} goes to a finite limit; the latter is usually smaller than 1, and becomes 1 for an extreme Kerr black hole ($a = M$).

For the equatorial plane, one can work out the energy of the product particles, say of D, in terms of: (1) the energies measured at infinity, E_A, E_B, and $E \equiv E_A + E_B$; (2) the angular momenta measured at infinity, L_A, L_B, and $L \equiv L_A + L_B$. Those also determine the collision region. One first goes to the locally nonrotating frame (LNRF) of Bardeen (1970; see also Bardeen *et al.* 1972) and then transforms, for simplicity, to a frame comoving with the center of mass (CMF). We shall denote by $\tilde{P}_{A\lambda}$ the momenta of particle A in the LNRF, and by $P_{A\lambda}$ the momenta of A in the CMF. We use similar notations for the other particles. We shall in particular need P_{A0}, which may be shown to be related to E_A and L_A by

$$P_{A0} = (E_A - \omega L_A)e^{-\nu} . \qquad (2)$$

The expression for E_D is now given in terms of S, the total energy in the CMF, by

$$E_D = \tilde{p}_{D0}\left(\frac{E}{S}\right) + \left\{\left[\left(\frac{E}{S}\right)^2 + \omega^2 e^{2\psi} - e^{2\nu}\right] \sum_k (\tilde{p}_{Dk})^2\right\}^{1/2} \Gamma \qquad (3)$$

with

$$S^2 = \mu_A{}^2 + \mu_B{}^2 + 2P_{A0}P_{B0} - 2\sum_k p_{Ak}p_{Bk} . \qquad (4)$$

* Part of this work was done while T.P. was with the Department of Environmental Sciences, Tel Aviv University.

Γ may vary between 1 and -1 and depends on E and L as well as on the spatial directions of motion. When near the horizon, it generally reaches its largest value when, in the CMF, particles C and D move (in opposite directions) along the $\hat{\phi}$ direction of the LNRF; one of them is then thrown into the most negative energy orbit possible.

As an example, consider the special case in which A falls in radially ($L_A = 0$) and encounters particle B which rotates around the black hole in a circular equatorial orbit. In the LNRF at the collision point, the spatial momenta are orthogonal, and, by (2),

$$P_{A0} = E_A e^{-\nu} \tag{5}$$

while, at the marginally stable orbit, say, for almost extreme Kerr,

$$P_{B0} = (E_B - \omega L_B)e^{-\nu} \sim \mu_B \,. \tag{6}$$

Consequently, near the horizon one has

$$S \sim (2E_A\mu_B)^{1/2}e^{-\nu/2}\,, \tag{7}$$

so that S increases beyond bounds. When scattered at the suitable angles discussed above, we obtain

$$E_D \sim \tfrac{1}{2}(E + S\omega e^{\psi}) \tag{8}$$

which tends to infinity as well. In other words, the energy gain here is virtually unbound.

Naturally, it is not immediately clear whether the particle carrying the high energy is, indeed, in an outgoing orbit; nevertheless, the fact that the energy gains are very large does suggest that the process will be more efficient than the three-body Penrose process. For example, let us impose the condition $p_D^{(r)} > 0$ on the four-body collision. A necessary condition for this to occur is

$$\kappa^2 \equiv 1 - \frac{\mu_D^2 p^2}{\tilde{p}_D^2 S^2}\psi\xi_2^2 > 0\,; \tag{9}$$

and, when this condition is satisfied, one finds

$$E_D < E_D^{\max} = E\frac{\tilde{p}_{D0}}{S}\left(1 + \frac{p^2}{S^2}\xi_2^2\right)^{-1} + (\omega e^{\psi}\mathcal{E} + e^{\nu}p\xi_1)\frac{\tilde{p}_D}{S}\kappa\left(1 + \frac{p^2}{S^2}\xi_2^2\right)^{-1}, \tag{10}$$

where the new symbols in (10) are:

$$p \rightarrow = p_A + p_B\,, \quad \mathcal{E} = p_{A0} + p_{B0}\,,$$
$$\xi_1 = \cos\phi\,, \quad \xi_2 = \sin\phi\,,$$
$$\phi = \text{angle between } \hat{e}^{(\phi)} \text{ and } p. \tag{11}$$

The angular momentum associated with $E_D^{\max}$ is found to be

$$L_D = L\frac{\tilde{p}_{D0}}{S}\left(1 + \frac{p^2}{S^2}\xi_2^2\right)^{-1} + e^{\psi}\frac{\tilde{p}_D}{S}\kappa\left(1 + \frac{p^2}{S^2}\xi_2^2\right)^{-1}\epsilon\,. \tag{12}$$

When one considers the Compton effect, letting A and D be photons with A falling in radially and having B rotate as before, one finds, on approaching extreme Kerr,

$$E_D^{\max} \rightarrow 2\mu_B/\sqrt{3}\,. \tag{13}$$

If the initial photon energy was

$$E_A < \mu_B/\sqrt{3}\,,$$

this means that particle C will have a negative energy, and a net energy gain has been obtained. Note, that because $r_{\rm ph} < r_{\rm ms}$ for every $a < M$, photon D will, indeed, escape to infinity.

In collisions of identical particles, such as p-p collisions, for which $p_D^{(r)} > 0$, one can see from (10) that no energy gain is obtained for our special case. However, $p_D^{(r)} > 0$ is not a necessary condition for having particle D on an outgoing orbit. A swift examination of (10) also reveals that high-gain Penrose processes, having $p_D^{(r)} > 0$, will also occur for $p \rightarrow 0$, i.e., for head-on collisions in the LNRF. This situation will exist in disk geometries, and its detailed analysis is now being carried out by us.

In order for the process to be effective, some amount of matter should be turning around very deep in the ergosphere and some amount should be accreting on the hole. We are thus performing detailed calculations of effective cross sections for real processes with "realistic" accretion rates in X-ray binaries, and we hope to discuss them in a separate publication.

Nevertheless, with a macroscopic geometry which is not physically unlikely, this could be a source of very energetic particles and may provide an additional way of extracting energy from a rotating black hole which is accreting matter.

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REFERENCES

Bardeen, J. M. 1970, *Ap. J.*, **162**, 71.
Bardeen, J. M., Press, W. H., and Teukolsky, S. A. 1972, *Ap. J.*, **178**, 347.
Boyer, R. H., and Lindquist, R. W. 1967, *J. Math. Phys.*, **8**, 265.
Kovetz, A., and Piran, T. 1974, submitted for publication.
Penrose, R. 1969, *Rev. Nuovo Cimento*, **1**, *Numero Speciale*, 252.
Wald, R. M., 1974, *Ap. J.*, **191**, 231.
Wheeler, J. A. 1970, *Proceedings of Vatican Conference on the Role of Nuclei in the Evolution of Galaxies*, 1970 April 15, as quoted by Bardeen, Press, and Teukolsky (1972).

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